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14. ABSTRACT
This report results from a contract tasking National Aerospace University as follows: High effectiveness of joints with transversal micro-fasteners, embedded in composite article, is proved experimentally in previous papers by the project manager. Project scope is working out structural and manufacturing solutions for metal-composite joints and correspondent of mathematical and experimental methods of engineering analysis. Following objectives are suggested to be considered in this project: (1) working out mathematical models and unified technique for determination stressed-strained state of joints with discrete, continuous and combined joining layer at arbitrary geometrical and rigidity parameters through the length of joining articles made of composites; (2) analysis of compliance coefficients influence on maximum stresses in articles and conformation of analysis formulas for determination compliance coefficients of micro-fasteners (0.8<d<2 mm), embedded in composite at the stage of its manufacturing; (3) working out the model of micro-fasteners interaction with composite reinforcement and obtaining formulas for determination composite elastic and strength properties in zones of curved reinforcement; (4) estimation of influence composite properties changing in zone of micro-fasteners embedding on stress distribution through joint length; (5) to prove that governing system of linear algebraic equations for stress determination in discrete joints has the same structure that Kirhgoff's law equations for electrical chains analysis. New simulation technique of joints modeling based on this suggestion will be suggested. This technique permits to model joint elements like electrical chains, that solve one of the most difficult problems for experimental research - force distribution through the joint length and between rows of fasteners; (6) mathematical formulas for determination similarity coefficients of joint mechanical properties and their electrical analogs will be obtained. Obtained results permit to solve important scientific and applied problem and ensure 5-30% savings of structure mass and workability.

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Final report

Joints of articles made of polymeric composite materials

Table of contents

List of figures

List of tables

Summary

The method of scientifically grounded joint type selection and joint parameters estimation is worked out. Influence of different external parameters on joints load-carrying ability is studied. Obtained nomograms permit to make decision about adhesive joint application or joints with micro-fasteners. Suggested technique permits to prove proper method of making holes for micro-pins installation. Estimated joint length can be criterion for joint mass calculation. Unified analysis scheme for joint elements stressed state determination based on physical discreetization approach is synthesized. Suggested analysis scheme considers structure and composition of joining layer and variable character of joining composite articles thickness and their physical and mechanical properties. Canonical solving systems of linear algebraic equations for determination forces in joint elements at joint loading both edges and arbitrary along joint length with axial, shear loads and thermal influence are derived. Proposed method gives ability for calculation compliance coefficients of joining layer and joining articles, estimate stress distribution through articles and joining layer at variable mechanical and thermal loading along joint length and takes into consideration like Poisson's ratio and thermal expansion influence. Comparison joint stress analysis as function on different used analysis schemes is conducted. Quite simple method for estimation joining layers components compliance confidence interval is suggested. Influence of embedded fasteners parameters on its own compliance and compliance of joined layer is grounded. Curvature of composite fibers due to micro-fasteners embedding is estimated and engineering dependences for evaluation changed parameters of composite article are suggested.

Introduction

Economical efficiency increasing is the criterion of new technique working out and manufacturing. That is why private and state firms of developed countries are conducted systematically forecasting researches devoted to revealing new tendencies and developing directions of different branches of technique.

According to conclusions of outstanding world scientific centers we can distinguish one main direction of aircraft efficiency increasing such as composites application in aircraft structure. Solving this problem influences significantly on aircraft structure mass reducing and increasing pay-load. Mass saving problem is the most severe one in aviation and cosmonautics branches. Each auxiliary kilogram of aircraft structure demands maintenance expenses up to $450...650 per year on each aircraft; launching on near-Earth orbit of one pay-load kilogram requires $5,000…10,000. Generally mechanical properties of metals and alloys have achieved their margin values so designers pay their attention to high-strength and high-modulus materials like composites.

More than forty years experience of composites application in aircraft structures proved that sharp mass saving can be exceeded in units with quite simple geometrical shape (rods, shells, etc) loaded with constant uniformly distributed loads (pressure vessels, rocket cases, fuel tanks etc).

Main load-carrying aircraft units (wing, fuselage, empennage etc) are generally withstand very complicated loading variable in time and space. That is why difficulties with proper composite reinforcement happen. The most undetermined cases appear in zones of force flow direction change (in so-called "zones of irregularities").

Aviation structures possess large variety of functional, maintenance and manufacturing joints, which are both exact irregularities zones and requires special material properties in joining zone (microhardness, wear-resistance etc). But level of these properties is quite low for majority of composites. That is why engineers forecast composites application in aircraft structure up to 75% only; last 25% cover different aircraft joints which withstand concentrated forces, fitting and joints with unavoidable metal articles.

Joints generally increase structure mass on 20% and cause approximately 80% of ruptures. That is why the problem of reliable joint design is very actual. From the point of view of composite nature it is better to use adhesive joints for their joining. But adhesive joint can transfer unit loads up to 1,5 kN/mm only at articles thickness about 2 mm.

Mechanical joint strength is 2…3 times lower comparing with metal ones so application very expansive composite is not very efficient.

Therefore main project idea is working out structural-manufacturing solutions of joints and relia-

ble techniques of undetachable and detachablejoint design which transfer high unit loads (more than 5kN/mm) and articles thickness of average and high thickness (more than 5 mm).

Section 1 Aircraft articles and assemblies joints review and analysis

Aircraft efficiency is stipulated significantly by its mass that plays main role on aircraft flight range, payload, fuel consumption and other operating characteristics. According to different literature sources to carry one auxiliary kilogram of Airbus mass they spend \$500 per airplane and to launch one auxiliary kilogram to near-Earth orbit it takes us \$5000…10000. Thus the most obvious way to increase aircraft efficiency is to reduce its mass. It is obvious that metal and alloys mechanical properties have achieved their margins so designers have to seek new structural solutions in the field of high-strength and high-modulus composites possessing highest specific strength and specific modulus ratio. That is why composites are spread widely in load-carrying schemes of up-to-date competitive airplanes, helicopters and other aircrafts. We can forecast that composites will be used in 75% of aircraft structure.

Problem of articles joining is the most actual for high-strength and high-modulus composite load-carrying aircraft structures. It is obvious that joints mass covers approximately 20% of entire aircraft mass. Moreover 80% of aircraft structure breakage begins from joining area. So proper joint type selection permits to use composites in load-carrying structure efficiently and as result to obtain high-efficient aircraft properties. By the way we have to note that there are quite strict limitations on joint structural solutions and their dimensions. These limitations are stipulated by:

– joint operational requirements;
– entire structure dimensions;
– requirements to aircraft aerodynamic surfaces;
– internal aircraft structure arrangement.

Aircraft joints analysis has shown that the most widely used aircraft joints are:

– mechanical ones (bolted, screwed, riveted);
– pure adhesive;
– combined adhesive- mechanical.

The most well-studied and trained are mechanical joints of metal articles. These joints ensure necessary value of structure load-carrying ability at defined reliability level. Technology of making holes for fasteners and fasteners installation to metal articles is routine one, quite simple and guarantees stability of design parameters. But it is impossible to apply conventional structural solutions of metals mechanical joining to composites without definite modifications. Application of mechanical fasteners in composites structures has revealed series of important problems for analysis:

– composites possess low bearing strength that is very important parameter at load transfer by contact article and fastener interaction;
– force flow in article/fastener contact zone has variable direction so composite structure has to be variable in this zone too;
– fastener hole causes high stress concentration factor;
– fastener body can be broken under variable loads;
– composite package can be damaged due fastener installation bearing stress;
– joining materials chemical interaction has to be analyzed thoroughly.

It is impossible to realize necessary level of load-carrying ability, joint lifetime and reliability without studying above-mentioned problems.

Joints investigations shown in source [1] has proved that all possible joints types have definite application range according to joining articles thickness or transferring load. Particularly mechanical joints are recommended for joining articles with thickness more than 1.7 mm:

– rivets can be used for thickness from 1.7 to 3.2 mm;
– bolts can be used for thickness more than 3.2 mm.

It was proved that economical efficiency coefficient reduces at joining articles thickness increasing for any joint type.

To realize high joint efficiency one has to study specific joining articles interaction processes.

These problems have been studied by many authors. Paper [2] analyzes joints parameters on composites contact (bearing) strength. So one can see that composite bearing strength reduces at fastener spacing less five fastener diameter and edge spacing less 2.5 fastener diameter. Empirical dependencies for bearing strength of some composites are suggested in the paper. But mentioned dependencies are not generalized ones and can't be applied for any composite. Source [3] analyzes composite/fastener interaction more thoroughly. Main conclusions of the paper are:

– coefficients of composite stress concentration are estimated;

– non-linear dependence of fastener interaction with contact hole area is proved, so contact area is to be variable one and depends on load value and installation fit force;

– angle of contact maximum (appr. 68°) corresponding to stable spot of contact is proved.

Paper [4] considers fastener/article interaction as combination of following processes:

– hole bearing;

– fastener shear;

– fastener bending;

– hole ovalization in zone of article loading.

The method of multi-row fasteners strength analysis has been worked out based on considered literature sources. Papers [5, 6, 7, 8, 9, 10, 11] analyze fastener/composite interaction modeling.

Analysis of theoretical and experimental data suggests ways for mechanical joints efficiency increasing:

– to increase composite bearing strength [12] one has to vary composite structure in local zone of contact (for example, to cover fastener hole with foil). But we should remember that local variation of composite structure causes its strength properties change;

– to remove problems of fastener body abrasion under broken fiber pieces paper [13] suggests to create holes for fastener installation at the stage of composite forming. The main drawback of this approach is local deviation of composite properties in zone of fastener installation. KhAI has conducted series of quite wide investigations devoted to properties of composites with installed micro-fasteners, so practical recommendations has shown in sources [14, 15] as result;

– application of cap-like fastener [16] with strong core permits to reduce radial installation stress and can be used in the case of single-side access. Possible variants of so-called “removable” fasteners (bolts or rivets) are worked out too;

– low-diameter fastener application (with diameter 1 mm appr.) are suggested in the paper [17]. This approach ensures proximity of composite load-carrying ability to load-carrying ability of joining articles;

– special techniques of load equalization between fattener rows have to be applied.

The last variant has to be considered more thoroughly. It is obvious that variable article thickness is quite efficient way for load spreading between fasteners [18]. But it is very difficult to adjust variable thickness article with neighboring ones (because machining is not the best way for laminated plastics adjusting). More convenient approach to this problem is to use fasteners with variable diameter, different fastener quantity in row and different fastener material. “Antonov Scientific and Research Corporation” has conducted similar experiments with positive results. Difficulties of this approach realization are stipulated by strict standards on possible fastener diameters. Paper [19] suggests using step-by-step replacing of composite load-carrying layers with metal sheets with the same thickness (at the beginning of joining zone metal sheets quantity is close to zero but in the middle of joint their fraction achieves 100%) and special fastener hole profiling. Special profiling means fastener hole shape and diameter machining in accordance with transferring load direction (i.e. hole diameter is slightly higher than bolt diameter in direction perpendicular to loading and hole diameter increases from the middle of joint to edges in loading direction). Gaps between bolt (rivet) and fastener hole surface is filled with randomly oriented reinforced material. Elasticity modulus of this filler in direction of force transferring is lower than main joining material has. This approach permits to equalize loading between fastener rows, to reduce stress concentration in weak section and solve problem of low bearing strength. The main drawback of this technology is necessity and complexity of local interlacing composite layers with metal ones. This drawback eliminating was considered in paper [20]. They suggest using cross-plied compo-

site as filler to compensate the gap between hole and fastener.

Adhesive joint is the most natural one for polymeric composites comparing with mechanical joint because of absence necessity of fastener holes drilling and load transferring through touching surfaces bearing. Thus, theoretically it is possible to join high-loaded articles by means of adhesion force but following problems of allowable level of joint load-carrying ability for large-volume production exist:

– very strict requirements to parameters and quality both preliminary article preparation and exact joining operation;

– absence of quite reliable non-destructive analysis methods in production process and in field conditions;

– repair impossibility in operation conditions;

– difficulties in polymer properties degradation forecasting (that is why majority of designers distrust joining high-loaded articles);

– possibility of composites delamination due to load transferring by lateral surfaces, low interlaminar shear modulus, low interlaminar shear strength.

A large amount of papers were devoted to adhesive joints analysis, we can distinguish following ones among them: O.Volkersen [21], A.L.Rabinovich [22], Goland-Reissner [23, 24], V.F.Kut'inov [25, 26] etc [27, 28]. These papers permit to forecast stress in adhesive layers theoretically but it is impossible to estimate this stress experimentally. So it very difficult to make well-defined conclusion about suggested estimation methods reliability. Single experimentally established fact is that adhesive layer failure begins from its edges. But this fact doesn't prove any suggested approach. From the other hand one has possibility to estimate adhesive joint quality by experimentally known joint load-carrying ability. This approach permits to use quite simple dependencies for estimating joint stress-strain state. We should to note that suggested dependencies are not very correct from the mechanics of solids point of view.

Special suggestions for adhesive joint load-carrying ability increasing have been worked out based on analyzed material. Paper [29] shows that there is joining articles minimal thickness which ensures necessary joint load-carrying ability (for articles with constant thickness) and joint load-carrying ability depends on joining articles loading sequence.

Experimental results and results of theoretical analysis show that generally the level of load-carrying ability per unit length doesn't exceed 1.5 kN/mm for considered joint types. Several structural and manufacture solutions (SMS) have been suggested to increase this load-carrying ability level. These SMS we can divide by two groups (at defined loading level and joining zone dimensions):

– solutions lead to equalizing load in adhesive layer;

– solutions lead to general level of stress reducing in adhesive layer.

First group of solutions can be realized by application of joint elements with variable geometrical parameters along joint length. It is obvious that most natural way is to vary joining articles thickness proportionally to operating loads similar to mechanical joints [18, 27, 29]. It is also possible to use adhesive layer with variable parameters [27]. Adhesive layer has to possess the highest compliance in zone of probable stress maximum and has to reduce compliance smoothly to zone corresponding to stress minimum. One has to note that full-scale realization of suggested SMS depends on very strict matching of joining articles rigidity and can't be realized practically. Practically so-called "truncated" solutions are used (articles thickness varies in the form of steps or piecewise linear joint rigidity variation) [18, 30, 31]. These solutions are not so effective but quite simple in practical realization.

Second group lead to general level of stress reducing in adhesive layer at defined loading level and joining zone dimensions are based on load transfer area increasing due to additional planes of shear (multiple lap joints). High-loaded double lap and double strap joints are widely used [32]. Transferring force flow is divided between several planes of shear reducing force flow intensity and compensating possible bending moments. Multiple-shear joints with shear planes quantity more than two permit to reduce stress in adhesive layer very significantly both increasing area of joining and decreasing articles rigidity between load transferring planes. Realization of above-mentioned solution in composite wing-spar structure permits to solve problem of thermal stress [33].

Important problem of interlaminar composite failure at the joint edge has been analyzed very tho-

roughly. The problem of interlaminar stress influence on adhesive joint load-carrying ability are distinctive generally for any laminated structure with compliant interlaminar connections (adhesive joint is representative of such structure). Bending moment adds its portion to interlaminar reaction [28, 34]. Profiled joint elements, like "scarf" joint, and application of multi-shear joint solution permit to reduce level of interlaminar stress but these solutions can't be applied for all cases or not so effective. The main restrictions in these solutions are technological ones. To prevent adhesive layer or composite article tearing (depending on weak chain) one can use transversal stitching of composite package with organic fibers or metal wire. Fibers withstand tearing (peeling) load and can increase composite elastic properties through article thickness at high density of stitching. Unique significant restriction is possibility of stitching both joining unpolymerized articles together or separately. Thus there are several unsolved in general form important problems of adhesive joining composite articles with metal fittings.

It is obvious that mechanical and adhesive joints possess both their advantages and drawbacks so one can use these joining types simultaneously [35]. It was assumed previously that exact fasteners withstand 75…80% of transferring loading and adhesive layer 20…25% only. But paper [36] shows that load-carrying ability of considered adhesive-bolted joint doesn't exceed load-carrying ability of pure adhesive joint, so glue carries the majority of load. Moreover this source proves that bolts introduce to load transfer after adhesive layer damage only or at presence of adhesive layer defect. Generally efficiency of combination of adhesives and fasteners together for increasing joint load-carrying ability is quite low and can be estimated by means of approach shown in [1]. But it is impossible to discard this approach at all. Paper [37] suggests:

– method for estimation strength and rigidity of combined (adhesive/bolted) joining layer with arbitrary arrangement;

– proves that load fractions transferred by bolts (rivets) and adhesive depend on their parameters ratio, for example, fastener diameter reducing corresponds increasing portion of its load-carrying ability in entire load-carrying ability of joint. Moreover the absolute level of joint load-carrying ability depends on proper parameters selection of adhesive/mechanical elements.

Conclusions

Necessity of composites application in high-loaded and critical aircraft structures demands effective and reliable solution of aircraft elements joining problem. Structural and manufacturing solutions of pure mechanical, pure adhesive and combined joints have been studied thoroughly. Following conclusions can be done based on analyzed materials:

1. Mechanical joints permit to transfer very high loads so they are very prospective. Main drawbacks of mechanical joints are load transferring by means of bearing contact surface. This fact demands unusual high bearing strength of composite material. Force flow variable direction near fastener zone needs in its turn composite stacking sequence correction in this local zone.

2. Main ways of mechanical joint efficiency increasing are:

– more smooth load distribution between fastener rows by means of fastener hole special profiling and variation joints parameters along joint length;

– adjustment of contact stress with ultimate composite bearing strength by means of composite stacking sequence variation and fastener diameter reducing;

Majority of structural and manufacturing solutions, ensuring high mechanical joint efficiency, generally, have low manufacturability or cause sufficient joint cost increasing.

3. Adhesive joint is the most suitable one for joining composites because of load transferring without contact surfaces bearing and elimination hole drilling operation for fasteners installation. The main drawback of adhesive joint is low load-carrying ability due to non-uniform stress distribution in adhesive layer and composite interlaminar stress influence on this distribution.

4. Main ways of increasing adhesive joint load-carrying ability are:

– special profiling of joining articles;

– joint variants with multi-shear planes application;

– joint transversal stitching application;

All these variants are directed to joint load-carrying ability increasing (like in mechanical joints) but cause cost increasing and can be used in definite structures only.

5. Combined adhesive/mechanical joints are not well studied yet. Preliminary experience of this type joint application shows its low efficiency (i.e. low load-carrying ability). The main advantage of this joint type is joint survivability increasing. Later explorations have shown that to increase joint load-carrying ability it is necessary to match joining elements parameters very thoroughly and mechanical fasteners with possible lowest diameters can be recommended.

From above-mentioned analysis we can make conclusion that conventional joint types exhausted their potential and their load-carrying ability increasing causes sufficient cost elevation. Analysis of theoretical and experimental data has shown that low-diameter fasteners application in pure state and in combination with adhesive is quite perspective. Working out of combined (mechanical/adhesive) joint type is the objective of this project.

Section 2 Structural and manufacturing solutions of joints with transversal micro-fasteners

Methods, Assumptions and Procedures

Analysis of questions devoted to designing articles made of composites and their applications in aircraft structures has shown that efficiency of **structural and manufacturing solutions** (SMS) depends on large amount of factors; the most important of them is proper selection of joints SMS.

Exactly unfounded way of load transferring causes overrated structure stress-strain state and untimely structure breakage. To avoid this phenomenon it is necessary to use large material quantity (by increasing safety factor value) that in its turn brings about over-weighted structure and decreasing efficiency of composite material application at all.

Load transferring method should be based on preliminary analysis. To conduct systemized analysis one have to use joints classification and criterion of definite joint solution selection. Joints classification should satisfy following requirements:

– it should cover existing and potentially possible joints SMS;

– classification should be continuous or has ability of stepped transition from one class to another;

– to have ability to predict future tendencies of objects developing and improving their characteristics;

– to compare efficiency of different classes;

– to estimate structural solutions quality

– should be quite clear, figurative and informative in SMS description.

Existing classifications based on maintenance principle, pure technological principle and principle of fastener type can’t satisfy above-mentioned requirements (excluding the fist requirement). National aerospace university (KhAI) has worked out new joints classification based on principle of force flow geometrical arrangement and control (so-called geometry of mechanical connections dislocation). Suggested classification satisfies above-mentioned requirements totally. Main parameters of this classification are shown in table 1.Таблиця 1 – Класифікація за принципом геометрії організації силових потоків

Table 1 – Joints classification by geometry principle of force flow distribution

Nb.	Joint class	Joint type (SMS distinctive feature)	Distinctive ways of load transferring

1	Discrete	Bolted, riveted etc. Point (spot) welded Flange joints etc.	Load transferring through discrete (singular) points
2	Linear	Seam (roller) welding Welding with longitudinal or lateral seams etc.	Load transferring through linear objects
3	Surface	Adhesive lap joint Soldered (brazed) Wedge joint etc.	Load transferring through article lateral sides (surfaces) only
4	Volumetric	Welding along side edge Adhesive joining along side edge etc.	Load transferring through article total cross-sectional area (trough any point)

Previous explorations conducted in KhAI have shown that decreasing exact joint mass in total structure mass is possible at transition from the first class to last one (at the same load-carrying ability).

According to this classification multi-row mechanical joints take position between fist and second class. Adhesive joints belong to the third class. Joints with transversal micro-fasteners occupy position between the third and the firth classes. Hence the highest joint efficiency can be achieved in joint with combination of adhesive and transversal micro-fasteners.

Criterion selection influences on effectiveness of SMS. Right now quality of SMS can be estimated by following three criteria:

– expenses on design, manufacture and operating;

– the value of auxiliary joint mass in aircraft structure;

– **workability coefficient**, which is equal to ratio of joint load-carrying ability to article load-carrying ability in regular zone.

It is quite difficult to use the first criterion at the stage of preliminary structural and manufacturing working out because of absence operating experience, lack of information about influence factors etc. A large amount of practical material has been gathered for prediction economical aspects of metal joints design; but for composite articles and units one has no enough practical information about their economical aspects.

Criterion of joint minimum auxiliary mass permits to estimate joint SMS efficiency at any stage of development and implementation. Moreover, having real values of operational costs on aircraft overweight maintenance, one can calculate potential economical consequences quite precisely.

Concerning to workability coefficient we can say that it is possible to join articles in such way that this coefficient can be equal to one or even can be more than one due to implementation of reinforcing, thickening etc. It means that practically we have no quite clear and established method to use this coefficient for estimation joint effectiveness. But if one can assume that joint shape, dimensions and structure are constant parameters workability coefficients fulfills role of quantitative characteristic of joining method. In this case the value of joint auxiliary mass is reversely proportional to **initial workability coefficient** value.

Introduction of notion of initial workability coefficient is quite acceptable for comparison joint load-carrying ability because of following reasons:

– initial workability coefficient is the direct result of general design and contains maximum information (articles dimensions, shape, structure, applied loading etc.);

– criterion of joint auxiliary mass reflects subjective and conventional information only;

– it is possible to compose table with quantitative characteristic of joint efficiency depending on joint SMS.

Moreover, the value of initial workability coefficient depends generally on stress concentration factor, which can be calculated by large amount of well-known theoretical and experimental methods.

Stress concentration factor can be conditionally divided by three components:

– k_1– stress concentration factor near discrete loaded point at assumption that stresses are uniformly distributed though article thickness;

– k_2– stress concentration factor near discrete loaded point due to non-uniform stresses distribu-

tion though joining articles thickness;

– k_3– stress concentration factor along multi-row, linear or surface joint.

First two coefficients characterize exact joint SMS but the third one depends on joint dimensions. The numerical value of this third coefficient doesn't depend on joint SMS and changes within range from 1 to infinity (at ratio of maximum stress to minimum stress).

This obstacle can prejudice estimation objectivity because of dominated value of k_3 in overwhelming majority variants of aircraft structure SMS. Hence, problem with coefficient k_3 proper determination can make application of initial workability coefficient criterion to be more complicated. This approach demands that:

– initial design data should be **the same**;

– or it is possible to compare load-carrying ability and dimensions (mass) of different SMS but **at the same value** of initial workability coefficient;

– or to compare joints load-carrying ability and k_3 value but **at the same joints dimensions**.

The first variant is more desirable because it can be used both developers and researchers. It was suggested to conduct joints comparison at $k_3 \to \infty$ to remove problem with proper k_3 selection. Joint load-carrying ability exceeds its possible limits and necessary joint dimensions (and auxiliary mass) exceed maximum at this assumption. Therefore **criterion of initial workability** can be modified to **criterion of limited initial workability**.

Analysis of **limited load-carrying ability** of mechanical and adhesive joints of identical articles (without considering operating temperature influence) has been conducted to establish degree of suggested criterion efficiency and work through its peculiarities. The joint model worked out in Central Aerohydrodynamic Institute has been used to define forces distribution between fastener rows. According to this model the system of equations adopted for this problem solution has following view:

$$N_{1,i-1}\Pi_c + N_{1i}\left(2\Pi - 2\Pi_c\right) + N_{1,i+1}\Pi_c = \begin{cases} N\left(\Pi + \Pi_c\right) \text{ at } i = 1 \\ N\Pi \text{ at } i > 1 \end{cases}, \quad i = 1\ldots n-1, \tag{1}$$

where N – total load, applied to joint; N_{1i} – load in the first article in zone between i and (i+1) row of fasteners; n – quantity of fastener rows; Π_c – force connection compliance; Π – articles compliances.

Solution of this system means such N values selection at which maximum stress in fastener doesn't exceed allowable value and minimum stress exceeds zero (practically it is more convenient to use value $\xi\tau_{\text{в fastener}}$, where ξ – predefined value, for example 0.01). Load per init length, stress in article, joint length, joining articles thickness are used for comparison joints load-carrying ability. $\tau_{\text{в fastener}}$ is used as allowable value in criterion of maximum stress. Restriction on stress level in fastener is defined by joint failure mode. Suggested method considers two main modes of joint failure: fastener shear and one of joining article bearing by contact surface. Taking into consideration other possible failure modes and structural and manufacturing restrictions doesn't influence on essence of analysis complicity.

Analytical dependence based on one-dimensional joint model has been used to estimate adhesive joint load-carrying ability [29]. For case of identical articles joining this dependence is the following

$$\overline{N} = \frac{2\tau_{\text{в adhesive}}}{k \,\mathrm{cth}\, \frac{kL}{2}}, \quad k = \sqrt{\frac{2\Pi}{\Pi_{jl}}}, \tag{2}$$

where $\overline{N}$ – limit of initial loading per unit length which defined adhesive joint can transfer; $\tau_{\text{в adhesive}}$ – adhesive shear strength; L – joint length; Π_{jl} – joining layer compliance, defined by Volkersen model.

Analysis has been conducted at following conditions (see table. 2–4).

Table 2 – Joining articles parameters

Parameters	Dimension	Value
Material elasticity modulus	GPa	50
Shear modulus in vertical plane	GPa	2.5

Poisson ratio	–	0.278
Article thickness	mm	0.5…5

Table 3 – Adhesive parameters

Parameters	Dimension	Value
Shear modulus	GPa	1
Adhesive thickness	mm	0.1
Adhesive sear strength	GPa	10; 20; 30; 40; 50

Table 4 – Mechanical joint parameters

Parameters	Dimension	Value		
Material elasticity modulus	GPa	100	200	
Fastener diameter, d	mm	1	1	3
Fastener bearing strength	MPa	400	400; 600	
Fasteners arrangement scheme	–	3dx3d	3dx3d 5dx5d	5dx5d
Allowable bearing stress	MPa	100; 200; 300		

Obtained results are shown as nomograms (fig. 1–5).

Results and discussions

Comparison of boundary surfaces for different joint characteristics permits to define the most effective field of joint application. Fig. 6-8 show application fields of different joints according to data defined on fig.1, 4, 5 (i.e. without considering material strength reduction due to holes presence).

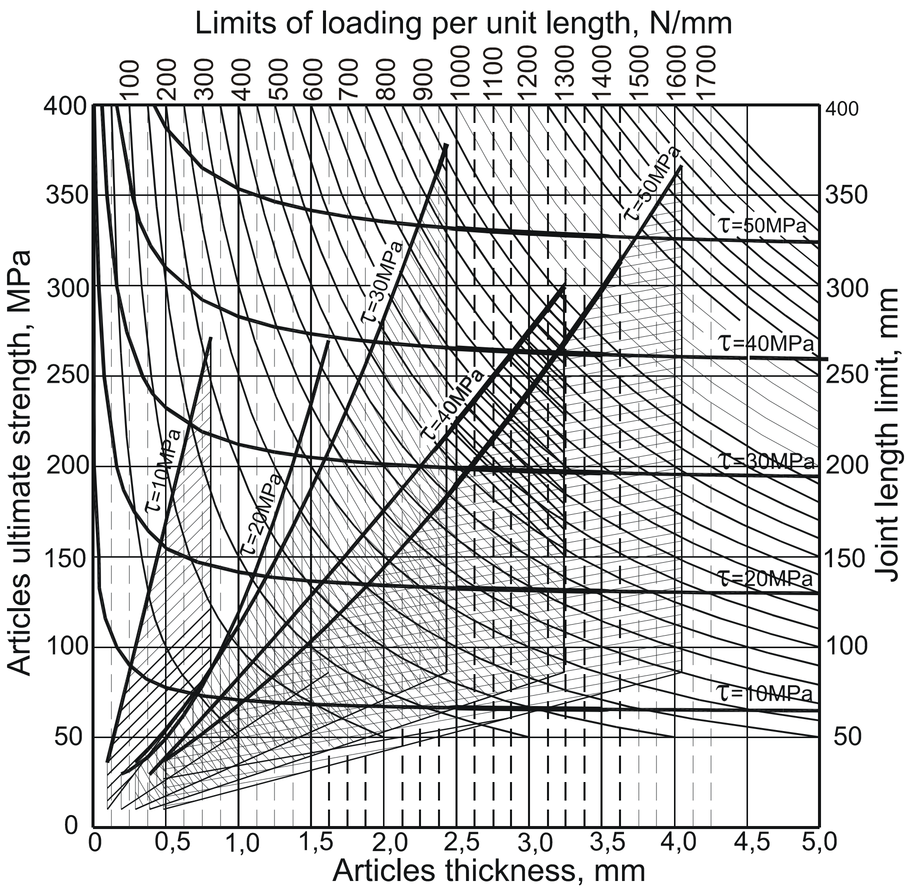


Fig. 1 Adhesive articles boundary surfaces

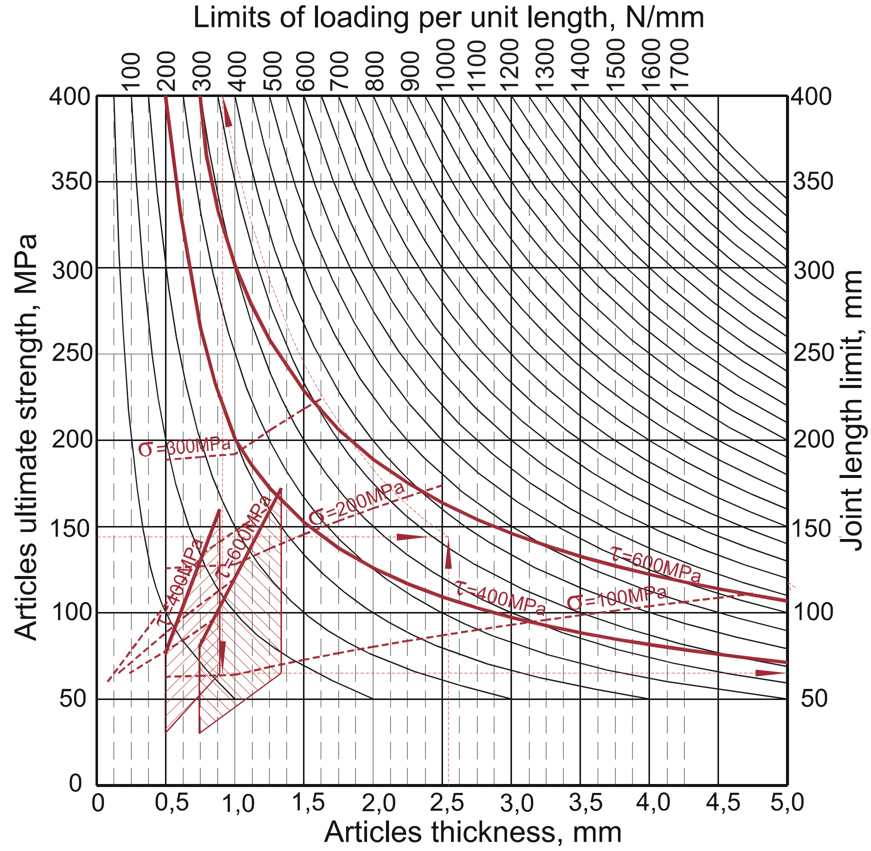


Fig. 2 Mechanical joint boundary surfaces ($E_{fastener}$=200 GPa, $d_{fastener}$= 1 mm, arrangement scheme – 5dx5d)

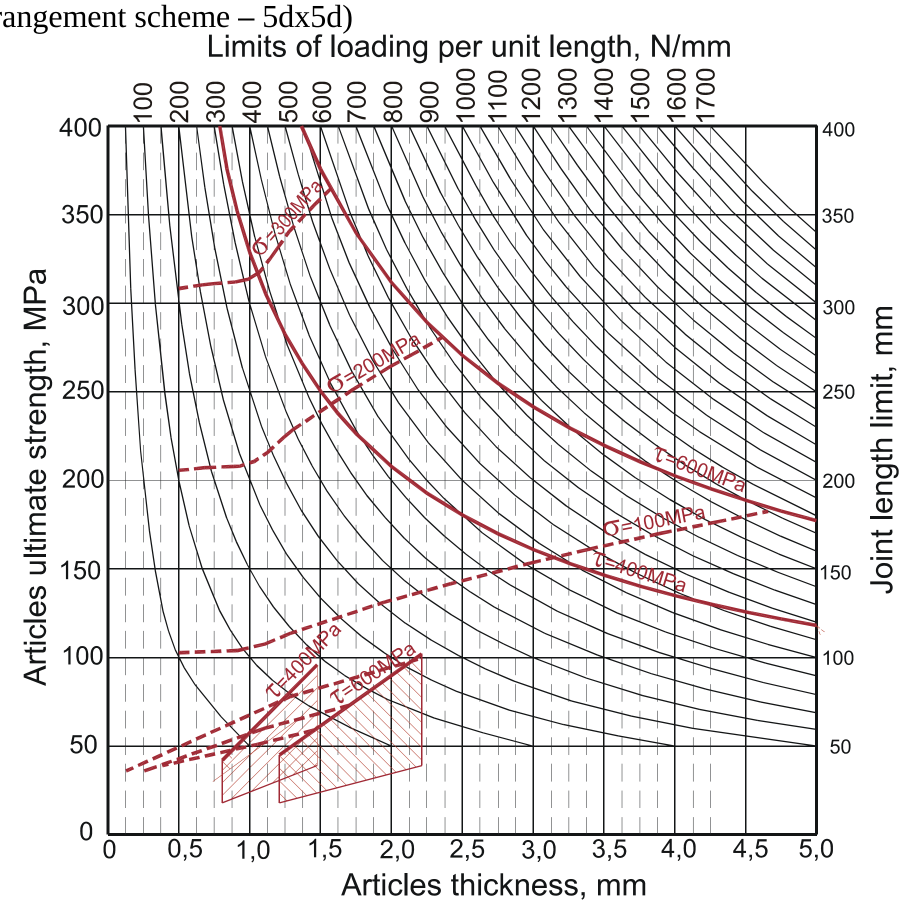


Fig. 3 Mechanical joint boundary surfaces ($E_{fastener}$=200 GPa, $d_{fastener}$= 1 mm, arrangement scheme – 3dx3d)

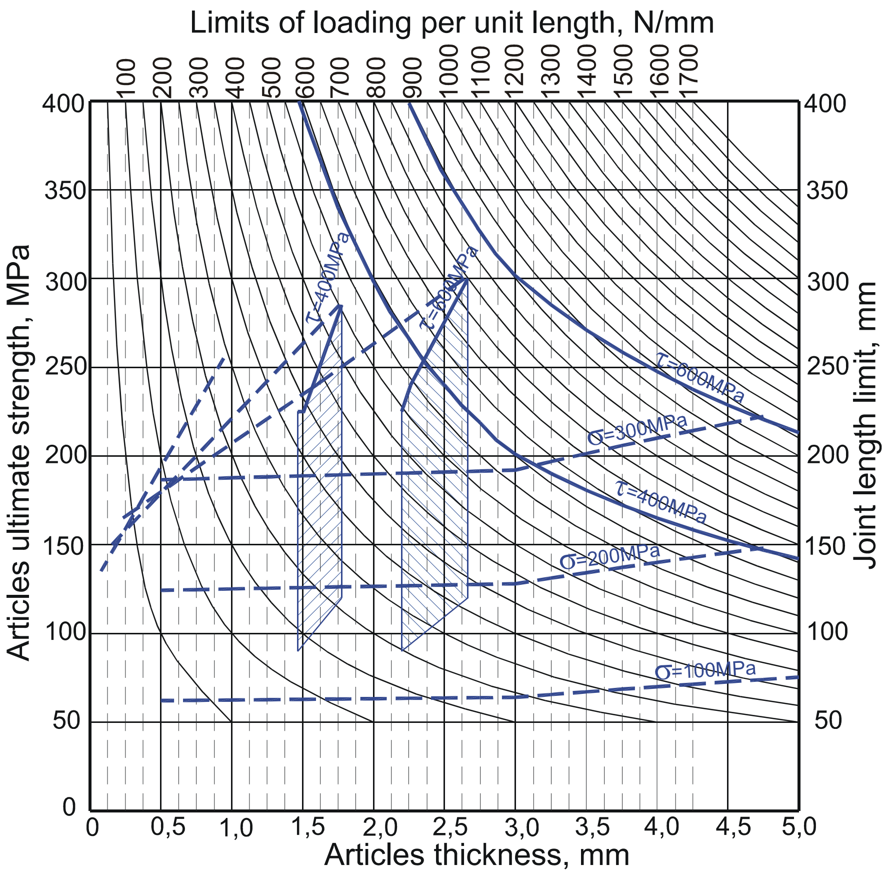


Fig. 4 Mechanical joint boundary surfaces ($E_{fastener}$=200 GPa, $d_{fastener}$= 3 mm, arrangement scheme – 5dx5d)

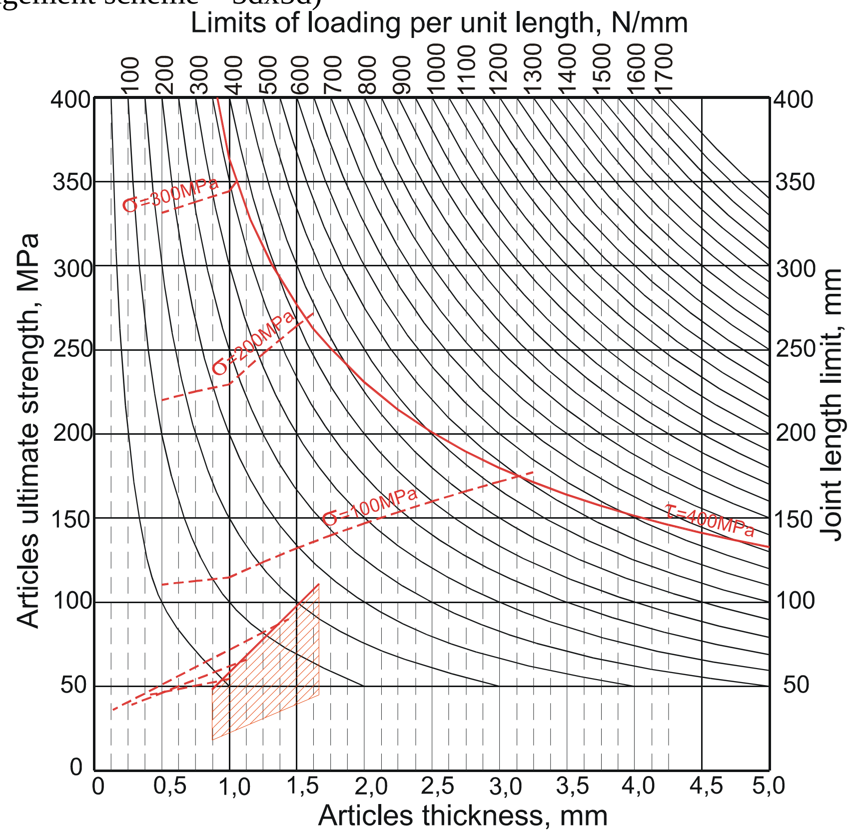


Fig. 5 Mechanical joint boundary surfaces ($E_{fastener}$=100 GPa, $d_{fastener}$= 1 mm, arrangement scheme – 3dx3d)

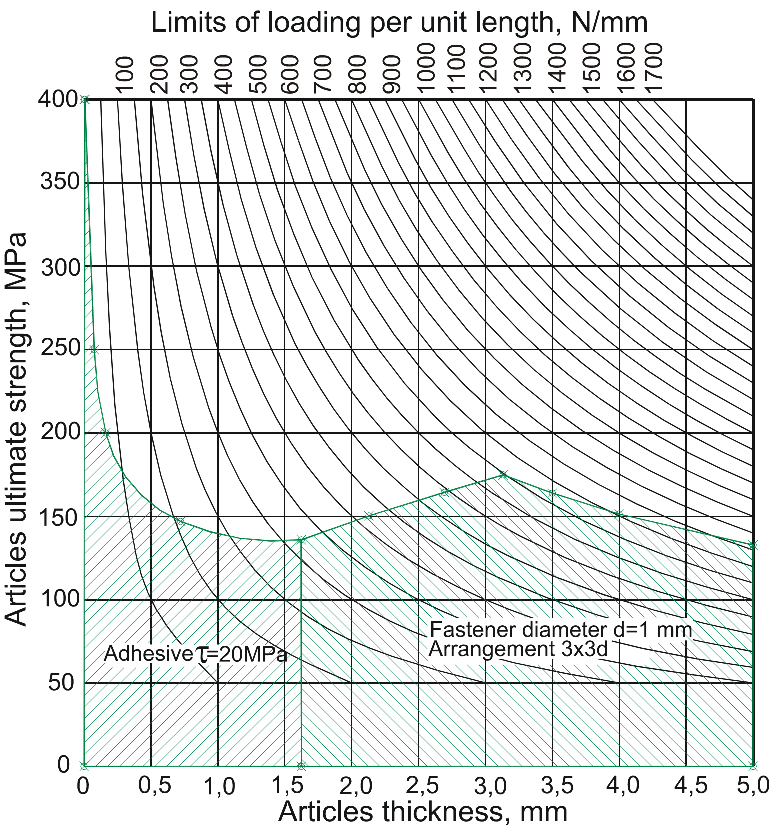


Fig. 6 Joint with micro-fasteners application field $\sigma_{\text{bearing}} = 100\,\text{MPa}$

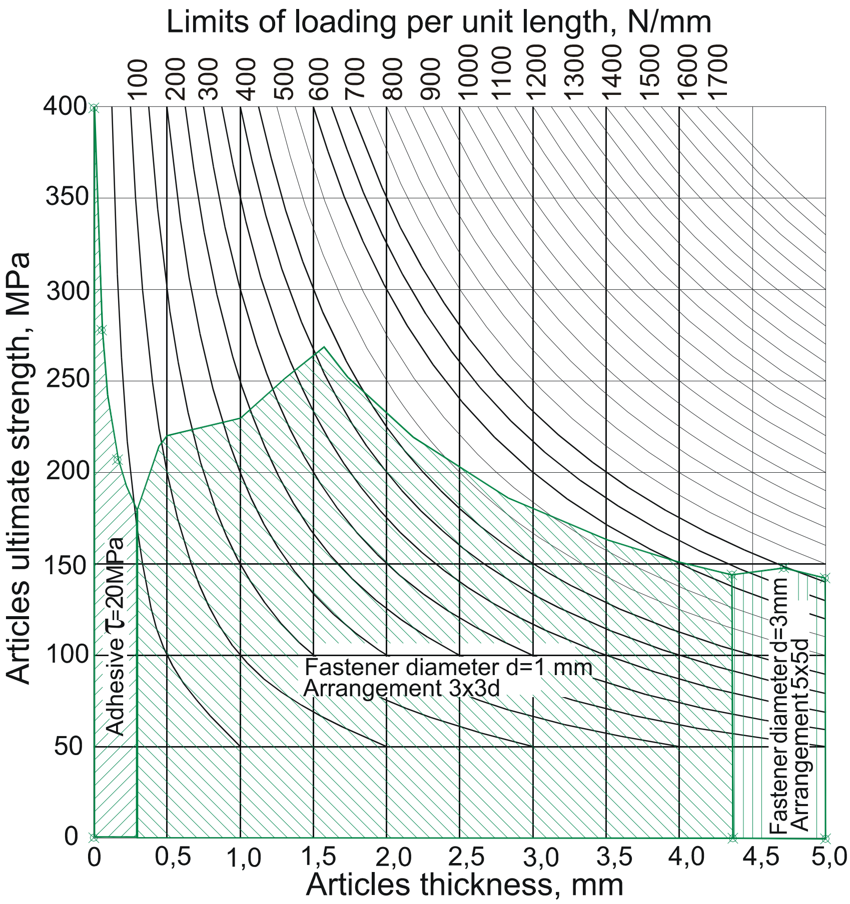


Fig. 7 Joint with micro-fasteners application field $\sigma_{\text{bearing}} = 200\,\text{MPa}$

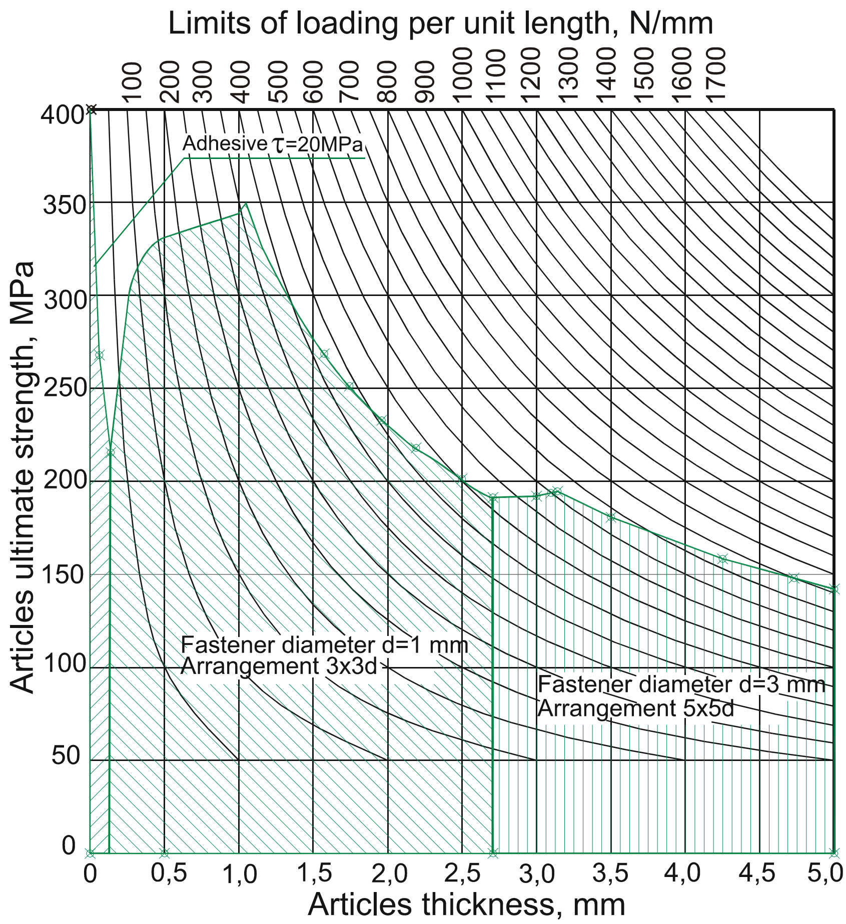


Fig. 8 Joint with micro-fasteners application field at $\sigma_{\text{bearing}} = 300\,\text{MPa}$

To satisfy condition of limited margin of stress near opening one can estimate the rate of joint load carrying ability reduction. We can use so called **weakening coefficient** k_{weak} to derive allowable stress level:

– for opening drilling

$$k_{weak} = \frac{t-1}{kt}, \tag{3}$$

where k– stress concentration factor near opening [38]; t– relative spacing of fasteners installation;

– for fasteners co-forming into wet composite according to [39]

$$k_{weak} = \frac{1}{2}\left(1+\left(\frac{3}{4t}\right)^2\right)^{-2} + \frac{2t}{3} arctg \frac{3}{4t}. \tag{4}$$

Analysis of above-mentioned fields considering expression (3) (at k=3) permits to derive following results (fig. 9–11).

Therefore we can see quite simple grounding method which permits to select proper joint type, to estimate joints parameters and to study influence of internal factors on joint load-carrying ability.

One drawback of suggested method was found at the stage of method approbation: predicted limited joint length is overestimated that can be explained by specific behavior of strapped joint. It means

that at $\overline{N} \rightarrow \overline{N}_{limit}$ $\partial L / \partial \overline{N} \rightarrow \infty$. But this drawback is not critical one and can be reduced if we will base our analysis on maximum stress but not on minimal one.

Thus we can estimate joining length based on predefined allowable overloading near 1% (see table 5 and fig. 9-11).

Table 5 – Joint length at 1% overloading of joining layer

Joint type	Article length, mm	Fastener diameter, mm	Installation scheme	$E_{fastener}$, GPa	Number of rows	Joint length, mm
Adhesive	0.5	—	—	—	—	10.2
	5	—	—	—	—	86
Mechanical	0.5	1	3x3	100	6	18
				200		
			5x5	200	6	30
		3	5x5	200	6	90
	5	1	3x3	100	15	45
				200	13	39
			5x5	200	13	65
		3	5x5	200	8	120

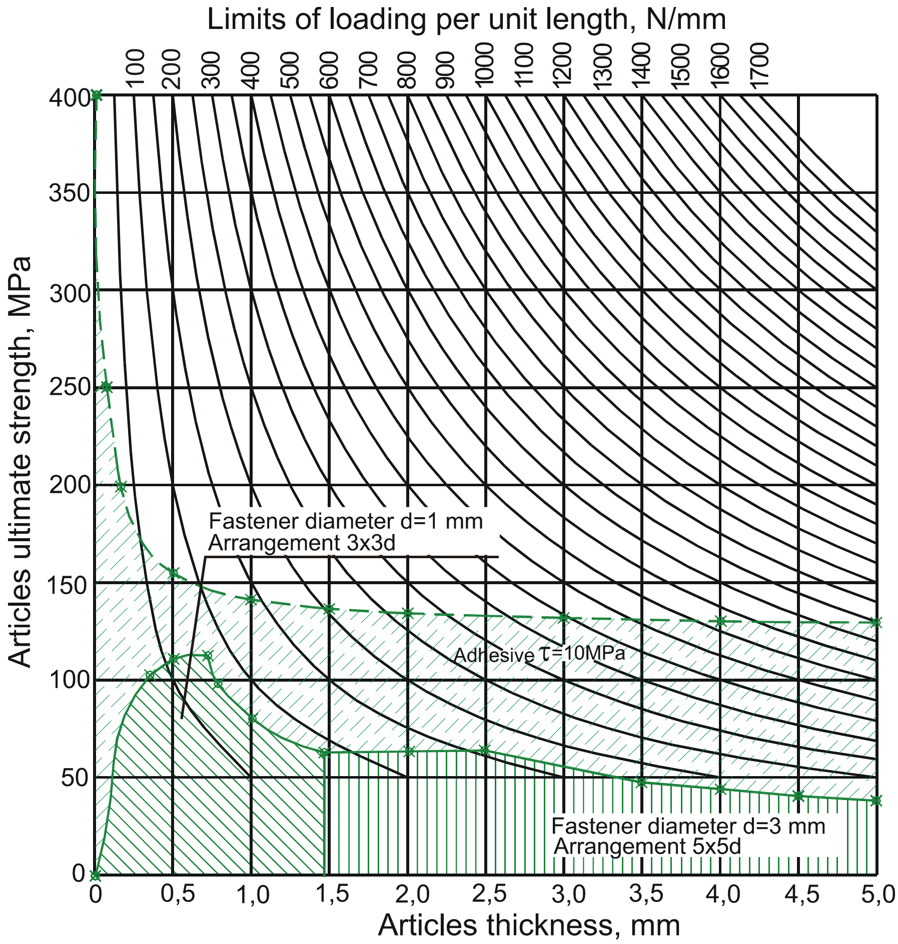


Fig. 9 Joint type implementation at $\sigma_{bearing} = 100\,\text{MPa}$

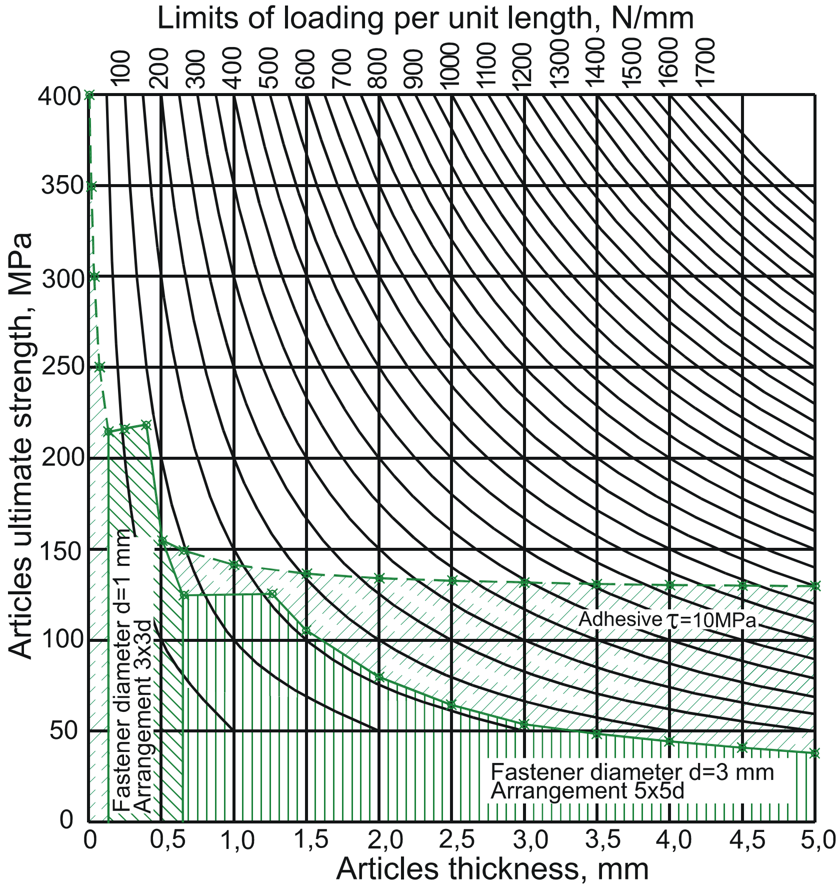


Fig. 10 Joint type implementation at $\sigma_{\text{bearing}} = 200\,\text{MPa}$

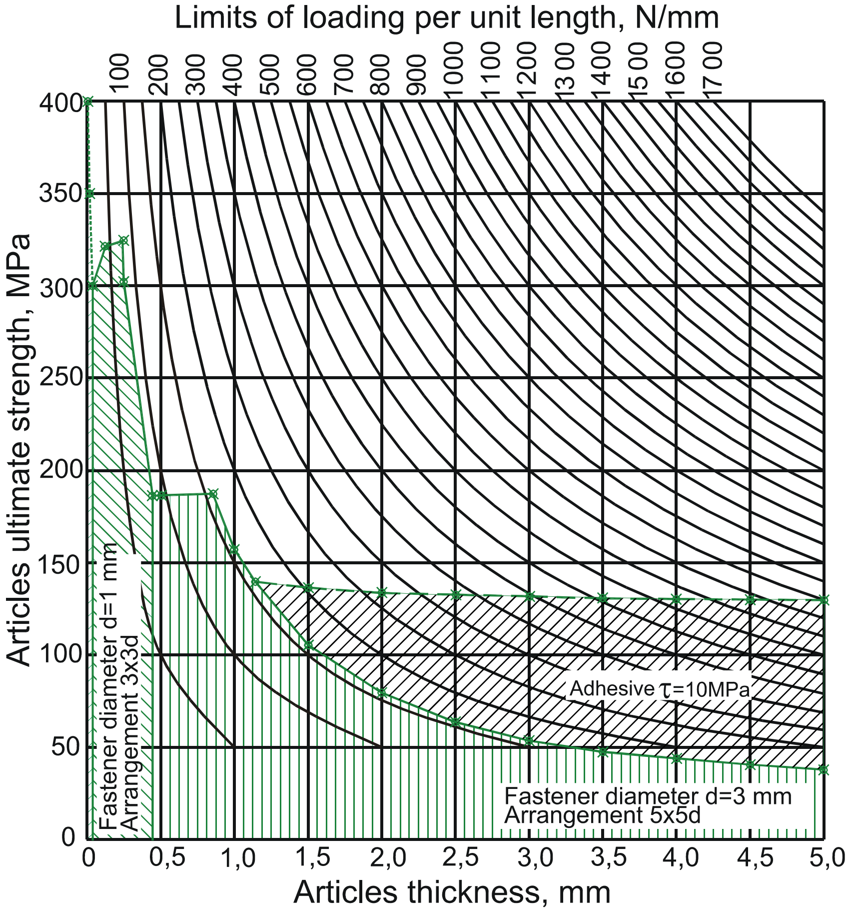


Fig. 11 Joint type implementation at $\sigma_{\text{bearing}} = 300\,\text{MPa}$

Conclusions

1. The method of joint quality estimation was worked out. The essence of this method is determination margin initial joint load-carrying ability considering different types of joint breakage: macro-fasteners shear, articles cross-section tear out, composite bearing.
2. Nomograms for pure adhesive and pure mechanical joints having different parameters of joining layer and composite bearing strength are composed (see fig. 1 – 5).
3. Distinctions of different mechanical joint failure modes influence on margin initial joint load-carrying ability.
4. Comparison of margin initial joint load-carrying ability was conducted and recommended application fields for considered joint types are defined.

Section 3 Synthesis of universal mathematical model for metal-composite joint stress state analysis

Methods, Assumptions and Procedures

Thus unified analysis scheme for joint elements stressed state determination based on physical discreetization approach is synthesized. Suggested analysis scheme considers structure and composition of joining layer and variable character of joining composite articles thickness and their physical and mechanical properties. Canonical solving systems of linear algebraic equations for determination forces in joint elements at joint loading both edges and arbitrary along joint length with axial, shear loads and thermal influence are derived. Proposed method gives ability for calculation compliance coefficients of joining layer and joining articles, estimate stress distribution through articles and joining layer at variable mechanical and thermal loading along joint length and takes into consideration like Poisson's ratio and thermal expansion influence.

The idea about physical discretization suggested in source [40] is main base of worked out joint **analysis scheme** (AS). A large amount of numerical examples have proved this approach for joint stressed state estimation guaranties practical coincidence with well-known analytical solutions at division structure on more than 20 elementary sections. Similar AS are widely used for mechanical (i.e. discreet) joints. This joint type possesses following advantages (as we mentioned in previous reports):

- character an frequency of division joint on sections can be easily adjusted with real micro-fasteners arrangement in joint;
- these AS are quite simple, that permits to use them at stage of joint tailoring;
- these AS ensure quite high precision for the stage of design because of estimation loads distribution between joint elements; this in its turn, permits to predict character and sequence of joint elements rupture;
- it is possible potentially to use **unique** AS for analysis all possible structural and manufacturing solutions (SMS) of lapped joints.

Moreover this AS can be implemented for joint analysis considering lateral loads applied through joint width (with several assumptions and restrictions).

Let's consider joint with variable parameters of joining articles and adhesive layer though joint length. Generally joint components are subjected by complicated thermal and mechanical loading (normal and tangent forces). Concentrated forces can be applied to joining articles both at joint edges and within joint elements (fig. 12). Theoretically quantity of applied forces is unlimited.

Suggested analysis scheme (like ones described in source [29]) is base on following **assumptions**:

- normal and shear stresses in articles (due to external forces and temperature)are distributed through article thickness uniformly;
- force connection (i.e. adhesive layer) carries shear stress only;
- bending moment (from normal external forces) and section twisting (from shear external forces) are out of consideration;
- geometrical shape of joining articles and adhesive layer composition along joint length can be

arbitrary (but should satisfy restrictions of manufacturability);

- adhesive layer composition and parameters of joining articles are constant in lateral direction.

In contrast to existing AS the system of external forces applied to joining articles can be divided on two groups: (1) making system (joint) to go out of equilibrium and (2) balancing (going in to the equilibrium). Thus, for example, forces of the first group can be applied to one of joining articles, forces of second group – to another joining article. Therefore, positive direction of external forces can be represented as shown at the fig. 13 and at the same time one should remember that positive stress in articles means article tension.

In progress of AS working out one can define real coordinate of external force resultant (fig 14, a) or spread load at points coinciding with nearest force connection rows (between which load is applied) (fig 14, b). No principal difference between these two variants for final results. But main difference in practical realization of these approaches is following: for first method one should add auxiliary nodes in discrete model; exact position of these nodes is referred with real load application coordinate, as result we obtain auxiliary solving equations (from equilibrium condition and compatible deformation condition), so procedure becomes more complicated (fig 14, c, e). For the second method discrete model is simpler (рис 14, b, d, f) and permits to create generalized unified AS for any joint analysis.

It is obvious that translating external force application point to nearest fasteners row will not influence on stress value in connection elements but causes stress distribution in correspondent article within unit division section.

Thus at this AS derivation one can match coordinates of possible load application points with coordinates of force connections to save AS simplicity. Maximum quantity of load application points is considered to be equal 2*n*, where *n* – force connections quantity. Force in connection is equal to zero at load absence at definite point.

It is possible to use internal forces differentiation (superposition principle) at complicated external loading in analysis process. Therefore it is suitable to consider separately AS of joint loaded with normal load and AS of joint loaded with shear load. Equilibrium equations for joint elements can be written as following for loading with normal forces only (under above-mentioned assumptions):

- for the fist article

$$N_{1i} = \sum_{j=1}^{i} P_{1j} - \sum_{j=1}^{i} Q_j ; \tag{5}$$

- for the second article

$$N_{2i} = \sum_{j=1}^{i} Q_j - \sum_{j=1}^{i} P_{2j} ; \tag{6}$$

- for force connections

$$Q_i = P_{1i} + N_{1,i-1} - N_{1i} , \tag{7}$$

where N_{1i}, N_{2i} - internal forces, acting in first and second articles correspondingly at *i*–th section;

P_{1i}, P_{2i} - external forces applied to *i*–th node in first and second articles correspondingly;

Q_i - internal force, taking by *i*–th row of force connections.

From equations (5) and (6) one can derive equilibrium of cut off joint part

$$N_{1i} + N_{2i} = \sum_{j=1}^{i} \left(P_{1j} - P_{2j} \right) . \tag{8}$$

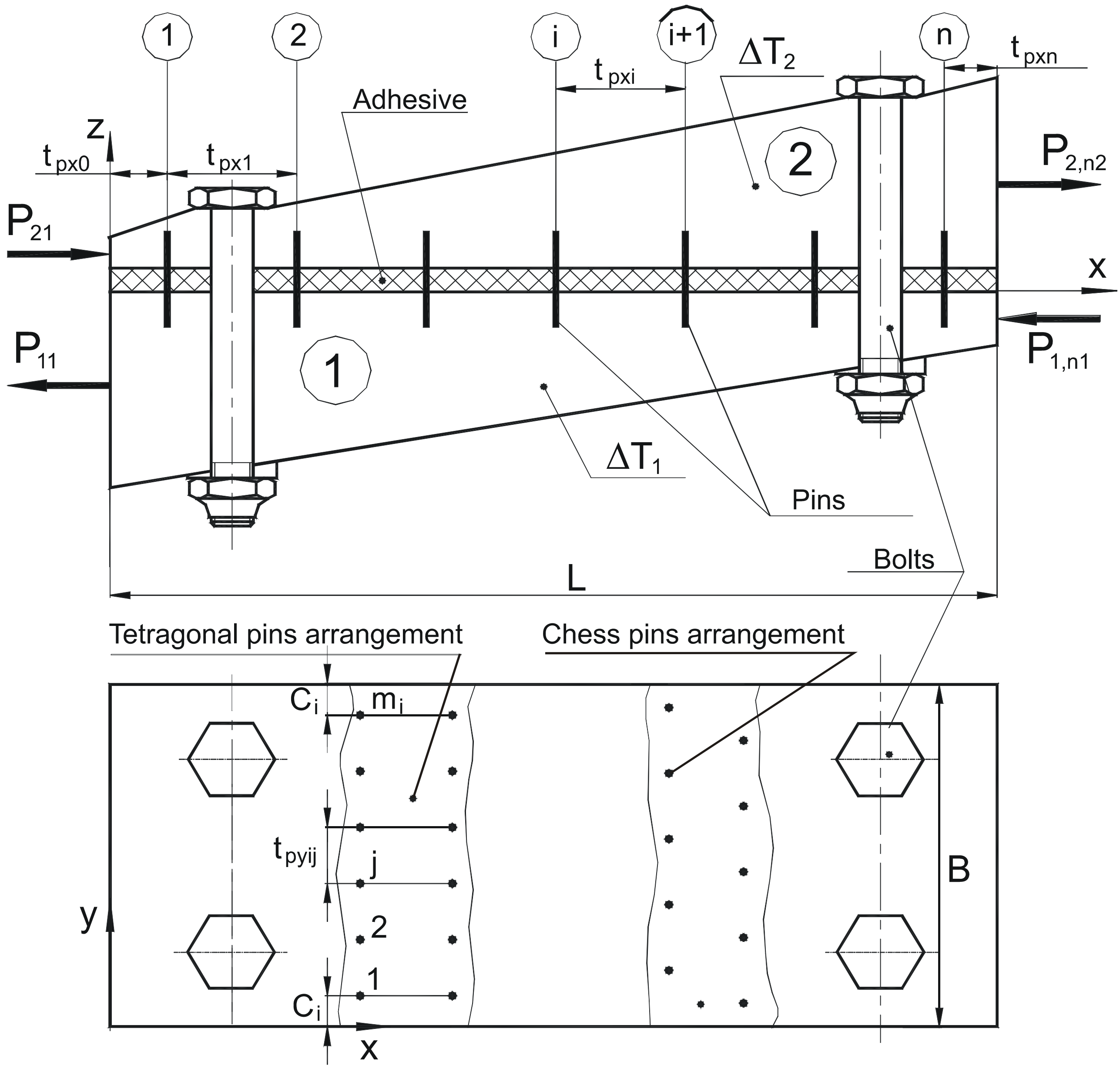


Fig. 12 – Lap joint model

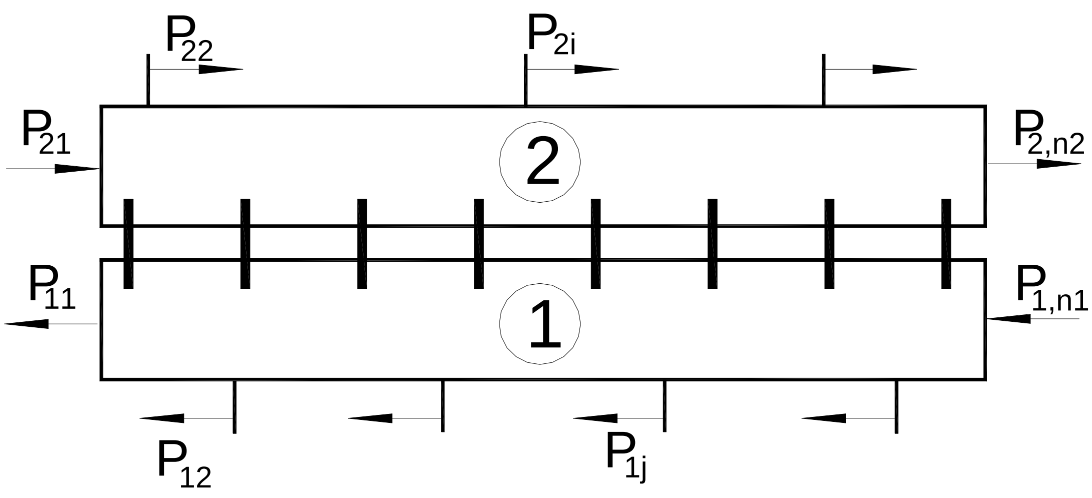


Fig. 13 – Positive direction of external forces application

Since synthesized AS is statically undefined we should use condition of compatible deformation (рис. 15, b) to obtain solving expression for our joint

$$Q_i \Pi_{3xi} + t_{xi}\left(N_{2i}\Pi_{2xi} + \alpha^*_{2xi}\Delta T_2\right) = Q_{i+1}\Pi_{3x,i+1} + t_{xi}\left(N_{1i}\Pi_{1xi} + \alpha^*_{1xi}\Delta T_1\right), \qquad (9)$$

where $i = 1..(n-1)$;

$\alpha^*_{1xi}, \alpha^*_{2xi}$ - average thermal linear expansion coefficients of articles material at definite section;

$\Delta T_1, \Delta T_2$ - temperature difference (is equal to difference between assembling temperature and operation temperature);

Π_{1xi}, Π_{2xi} - articles compliance along x axis (along joint length);

Π_{3xi} - compliance of force connections row.

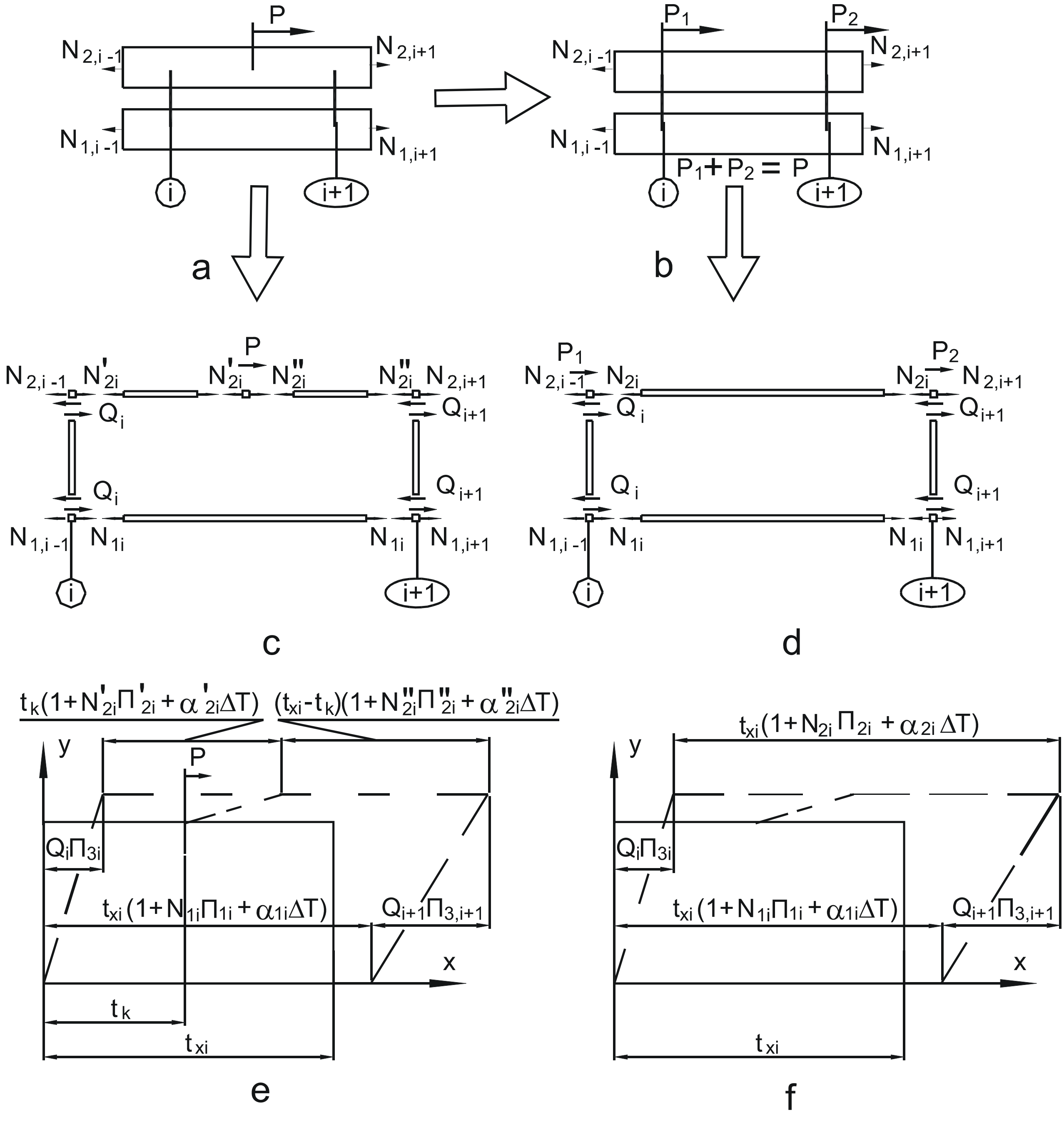


Fig. 14 – Possible variants of joint modeling in zone of concentrated load application

After definite transformations system (1.5) can be written as:

$$-N_{1,i-1}\frac{\Pi_{3xi}}{t_{xi}}+N_{1i}\left[\Pi_{1xi}+\Pi_{2xi}+\frac{1}{t_{xi}}\left(\Pi_{3xi}+\Pi_{3x,i+1}\right)\right]-N_{1,i+1}\frac{\Pi_{3x,i+1}}{t_{xi}}=$$
$$=\Pi_{2xi}\sum_{j=1}^{i}\left(P_{1j}-P_{2j}\right)+P_{1i}\frac{\Pi_{3xi}}{t_{xi}}-P_{1,i+1}\frac{\Pi_{3x,i+1}}{t_{xi}}-\beta_{xi}, \quad i=1...(n-1), \tag{10}$$

where

$$\beta_{xi}=\alpha^{*}_{1xi}\Delta T_1-\alpha^{*}_{2xi}\Delta T_2. \tag{11}$$

Thereby we obtained solving system of linear algebraic equations for determination internal forces in the first article (10), joining layer (7) and in the second article (8) or (6). Entire system of solving contains $2n+(n-1)$ equations for determination $2(n-1)+n$ variables, i.e. one equation is linear-dependent and describes equilibrium of entire system (joint).

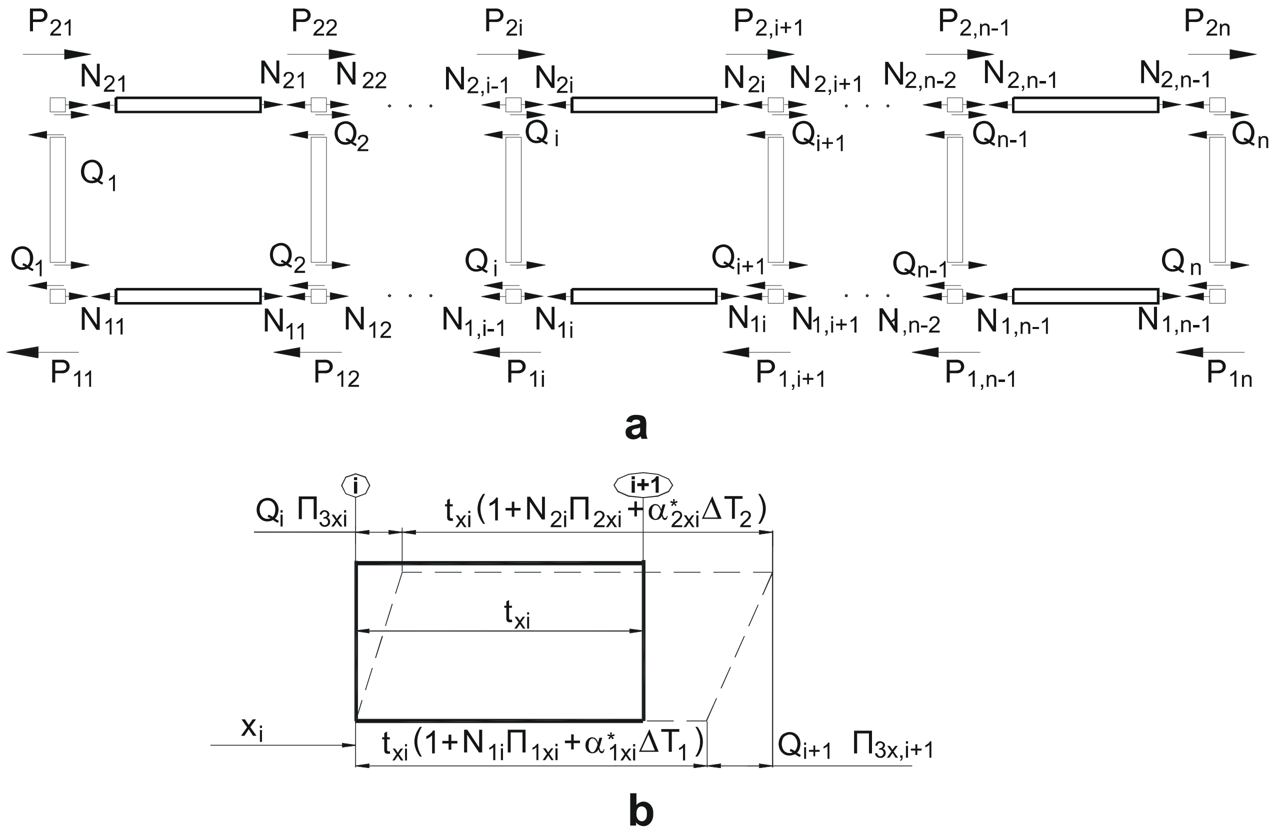


Fig. 15 – Joint discrete model

Quantity of fasteners or discrete force connections in lateral direction is more than one. That is why self-balanced stressed state of articles and fasteners appears in lateral direction at longitudinal loading due to Poisson's ratio and thermal deformations influence. It was proved in reference [41] that proper solution of the problem is very complicated and lengthy, therefore to obtain approximate solution of the problem we suggest to use above-mentioned unidirectional model.

To define loading components along y-axis method proposes selection arbitrary row of force connections (fig. 16) and creation discrete shear points considering fasteners installation arrangement (tetragonal – fig. 16, a or chess – fig. 16, c). Equilibrium equations for article sections have following form (рис. 16, b, d)

$$N_{1yij}=-\sum_{j=1}^{j}Q_{yij}; \quad N_{2yij}=\sum_{j=1}^{j}Q_{yij}; \quad j=1...m_i. \tag{12}$$

Forces in connections can be estimated by means of formula

$$Q_{yij}=N_{1yi,j-1}-N_{1yij}. \tag{13}$$

Compatibility of article deformation and connection deformation is described by expression (fig. 16, e)

$$N_{1yij}\Pi_{1yi}-N_{2yij}\Pi_{2yi}=\frac{1}{t_{yij}}\left(Q_{yij}\Pi_{3yij}-Q_{yi,j+1}\Pi_{3yi,j+1}\right)-\beta_{yi}, \tag{14}$$

where $j=1..\left(m_i-1\right)$;

$$\beta_{yi}=\alpha^*_{1yi}\Delta T_1-\alpha^*_{2yi}\Delta T_2-\varepsilon^*_{1yi}+\varepsilon^*_{2yi}; \tag{15}$$

$\varepsilon^*_{1y},\varepsilon^*_{2y}$ – average Poisson's deformations for exact section along y-axis from loading along x-axis.

$$\varepsilon^*_{1yi}=\frac{1}{t^*_{xi}B}\left[N_{1,i-1}\int\limits_{x_i-\frac{t_{x,i-1}}{2}}^{x_i}\frac{\mu_{1xy}(x)dx}{\delta_1(x)E_{1x}(x)}+N_{1i}\int\limits_{x_i}^{x_i+\frac{t_{xi}}{2}}\frac{\mu_{1xy}(x)dx}{\delta_1(x)E_{1x}(x)}\right];$$

$$\varepsilon^*_{2yi}=\frac{1}{t^*_{xi}B}\left[N_{2,i-1}\int\limits_{x_i-\frac{t_{x,i-1}}{2}}^{x_i}\frac{\mu_{2xy}(x)dx}{\delta_2(x)E_{2x}(x)}+N_{2i}\int\limits_{x_i}^{x_i+\frac{t_{xi}}{2}}\frac{\mu_{2xy}(x)dx}{\delta_2(x)E_{2x}(x)}\right].$$

Equilibrium conditions (12) and (13) can be inserted to (14) equation and we can obtain solving system of equations for determination internal forces N_{1yij}.

$$\begin{gathered}-N_{1yi,j-1}\frac{\Pi_{3yij}}{t_{yij}}+N_{1yij}\left[\Pi_{1yi}+\Pi_{2yi}+\frac{1}{t_{yij}}\left(\Pi_{3yij}+\Pi_{3yi,j+1}\right)\right]-\\-N_{1yi,j+1}\frac{\Pi_{3yi,j+1}}{t_{yij}}=-\beta_{yi},\quad j=1...(m_i-1).\end{gathered} \tag{16}$$

System (16) together with dependencies (12), (13) permits to define internal forces in lateral direction at joint loading with normal forces and thermal field influence.

A large variety of aviation structure joints frequently carry shear forces (for example wing spar cap and web joint, skin-rib joint, skin-bulkhead joint etc.). That is why engineers have necessity in design method of joint loaded with shear forces, assuming that:

– dimension B is quite large;

– structural measures exclude reciprocal articles twisting refer normal axis;

– joining articles have closed form (for example, shells).

These assumptions permit to consider this loading case using above-mentioned approach.

Equilibrium equations can be written like in case of joint normal loading (see fig. 1.4, b, c):

$$q_{1i}=q_{10}-\sum_{i=1}^{i}P_{yi};\qquad q_{2i}=q_{20}+\sum_{i=1}^{i}P_{yi}, \tag{17}$$

Assuming that articles material is orthotropic in plane x-y we can compose equation of articles compatible deformation at the section between two neighboring force connection rows (fig. 17, c):

$$t_{xi}q_{2i}\Pi_{2xyi}+P_{yi}\Pi_{3xyi}=t_{xi}q_{1i}\Pi_{1xyi}+P_{y,i+1}\Pi_{3xy,i+1}, \tag{18}$$

where $i=1..\left(n-1\right)$;

P_{yi} – force transmitted by *i*-th row;

Π_{1xyi}, Π_{2xyi} – shear compliance of articles in their plane (the method of shear compliance determination is considered further);

Π_{3xyi} – compliance of force connections row along y-axis.

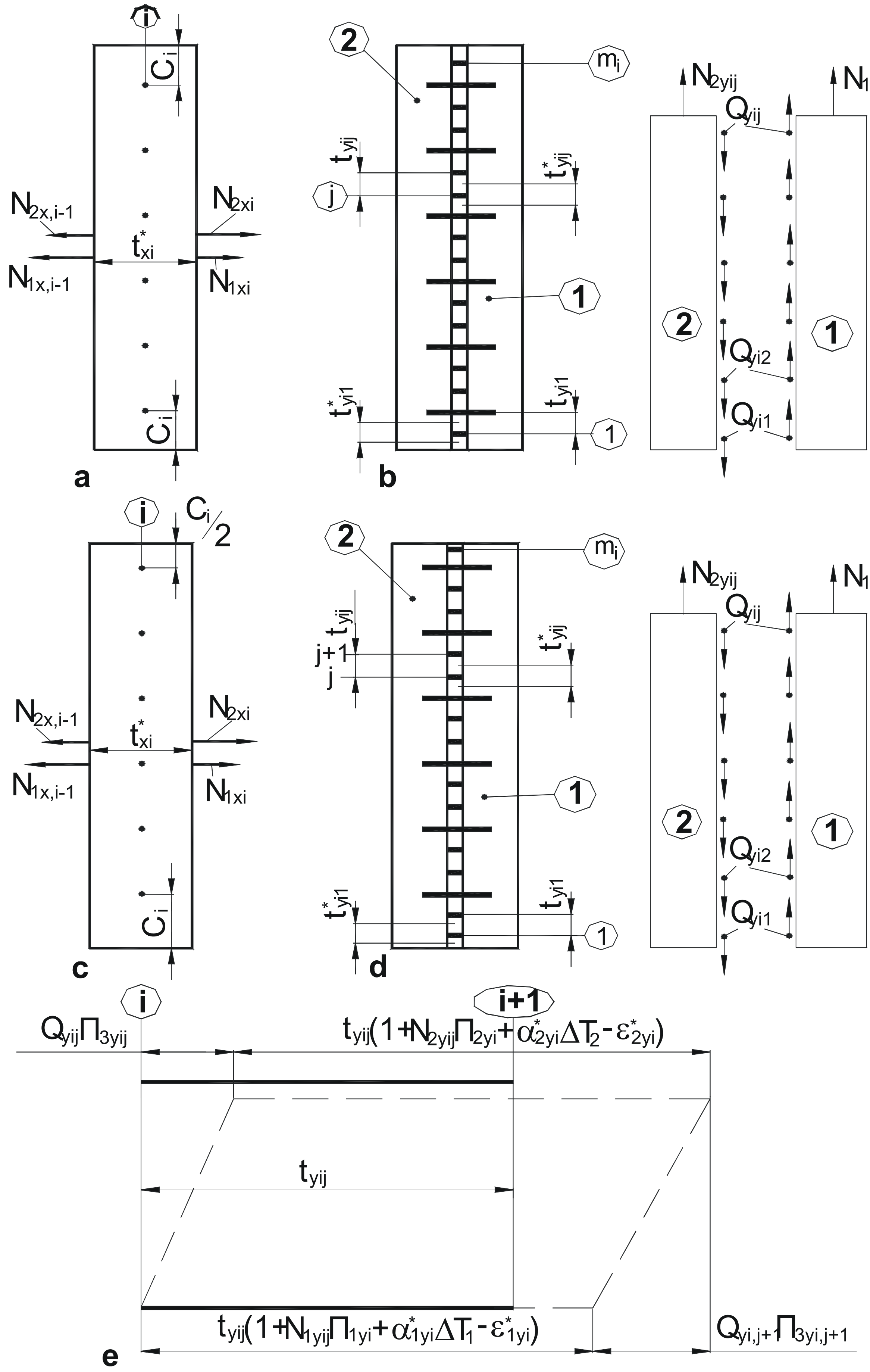


Fig. 16 – Joint discrete model in lateral direction

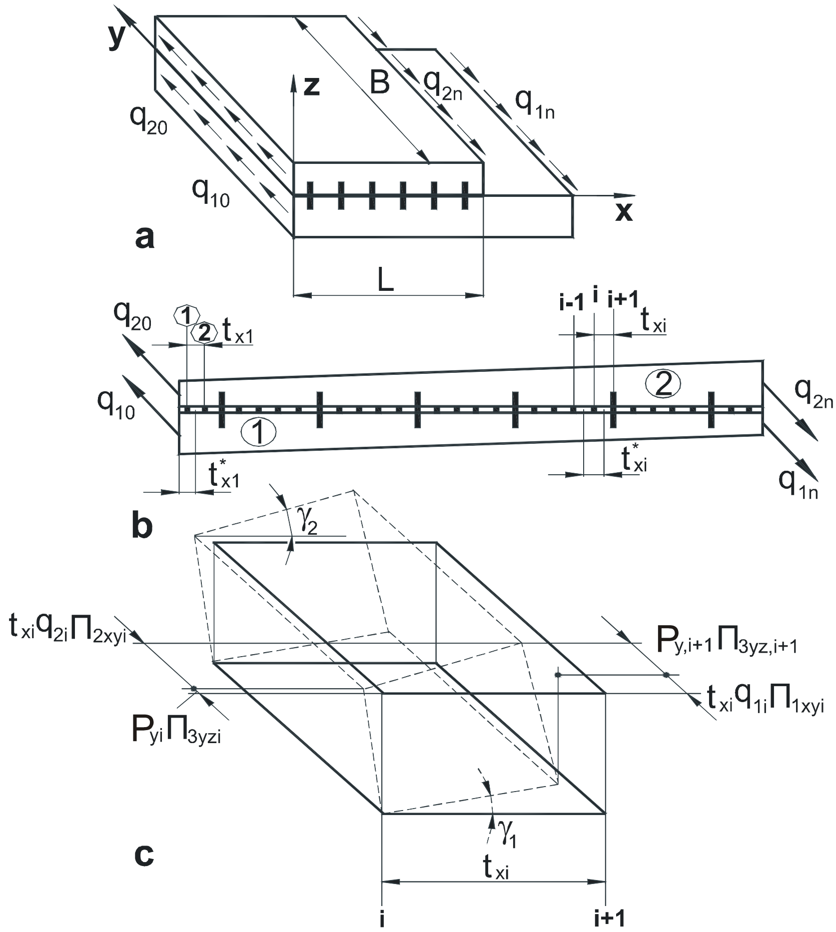


Fig. 17 – Joint model at shear

Forces in joining layer P_{yi} are defined from equilibrium condition (17) by means of internal forces in articles

$$P_{yi} = q_{1,i-1} - q_{1i} = q_{2,i-1} - q_{2i}. \tag{19}$$

Internal forces in second article is derived from equilibrium condition (17) by means of q_{1i} forces

$$q_{2i} = q_{10} + q_{20} - q_{1i}. \tag{20}$$

Inserting (19) and (20) expressions to system (18) and making transformations one can obtain resolving system of equations for determination forces q_{1i}:

$$\begin{aligned} -q_{1,i-1}\frac{\Pi_{3xyi}}{t_{xi}} + q_{1i}\left[\Pi_{1xyi} + \Pi_{2xyi} + \frac{1}{t_{xi}}\left(\Pi_{3xyi} + \Pi_{3xy,i+1}\right)\right] - \\ -q_{1,i+1}\frac{\Pi_{3xy,i+1}}{t_{xi}} = \Pi_{2xyi}\left(q_{10} + q_{20}\right); \quad i = 1...(n-1). \end{aligned} \tag{21}$$

Results and discussion

Thus unified analysis scheme for joint elements stressed state determination based on physical discreetization approach is synthesized. This AS naturally considers structure and composition of joining layer and variable character of joining composite articles thickness and their physical and mechanical properties (composite permits to vary both thickness and its physical and mechanical properties). Canonical solving systems of linear algebraic equations for determination forces in joint elements at joint loading both edges and arbitrary along joint length with axial, shear loads and thermal influence are derived.

Section 4 The method of determination joint components compliance coefficients

4.1 Joining articles compliance

Methods, Assumptions and Procedures

Last section considered approach for determination forces in joint elements through compliance coefficients and thermal linear expansion coefficients. It is obvious that variable geometry and structure of joining composite articles along joint length stipulate taking into consideration dependence of compliance coefficients and thermal linear expansion coefficients on these variable parameters too.

Physical meaning of joining articles compliance coefficients Π_{1x}, Π_{1y}, Π_{1xy}, Π_{2x}, Π_{2y}, Π_{2xy} is average deformation of article section with length t_x under unit load. In real structures articles thickness generally varies along x-axis, so let's consider derivation of necessary dependencies for generalized case of variable geometrical and rigidity parameters [29].

Article elongation on *i*-th section (see fig. 15, b) at $N_{1xi}=N_{2xi}=q_{1i}=q_{2i}=1$ equals

$$\Delta=\frac{1}{B}\int_{x_i}^{x_i+t_{x_i}}\frac{dx}{E_{1x}(x)\delta_1(x)}\ .$$

Then average strain (i.e. compliance) can be expressed

$$\Pi_{1xi}=\varepsilon_{average}=\frac{\Delta}{t_{x_i}}=\frac{1}{Bt_{x_i}}\int_{x_i}^{x_i+t_{x_i}}\frac{dx}{E_{1x}(x)\delta_1(x)}\ . \tag{22}$$

Similarly (see fig. 17, c)

$$\Pi_{2xi}=\frac{1}{Bt_{x_i}}\int_{x_i}^{x_i+t_{x_i}}\frac{dx}{E_{2x}(x)\delta_2(x)}\ ;\quad \Pi_{1xy}=\frac{1}{Bt_{x_i}}\int_{x_i}^{x_i+t_{x_i}}\frac{dx}{G_{1xy}(x)\delta_1(x)}\ ;$$
$$\Pi_{2xy}=\frac{1}{Bt_{x_i}}\int_{x_i}^{x_i+t_{x_i}}\frac{dx}{G_{2xy}(x)\delta_2(x)}\ . \tag{23}$$

To define article compliance coefficients in lateral direction one should consider deformation of element from $x_i-t_{x,i-1}/2$ to $x_i+t_{xi}/2$ under load N_{yij}. Since article rigidity within considering section can be variable (fig. 18) – $E_{1y}(x)$, $\delta_1(x)$ that average elongation of article section can be defined by formula

$$\Delta_{average}=\frac{N_{1yij}}{\left[E_{1y}\delta_1\right]_{average}}, \tag{24}$$

where, $\left[E_{1y}\delta_1\right]_{average}=\frac{1}{t^*_{xi}}\int_{x_i-t_{x,i-1}/2}^{x_i+t_{xi}/2}E_{1y}(x)\delta_1(x)dx,\qquad t^*_{xi}=\frac{1}{2}\left(t_{x,i-1}+t_{xi}\right).$

Average deformation is calculated as

$$\varepsilon_{average}=\frac{\Delta_{average}}{t_{yij}}=\frac{N_{1yij}}{\left[E_{1y}\delta_1\right]_{-p}\ t_{yij}},$$

which can be transformed (after several manipulations) to following expression (at $N_{1yij}=1$):

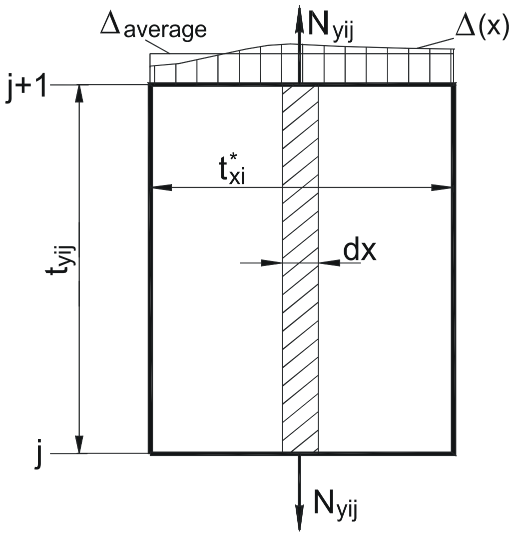


Fig. 18 Scheme for determination compliance along *y*-axis

Results and Discussions

$$\Pi_{1yij}=\frac{t^*_{xi}}{t_{yij}}\left[\int\limits_{x_i-t_{x,i-1}/2}^{x_i+t_{xi}/2}E_{1y}(x)\delta_1(x)dx\right]^{-1}. \quad (25)$$

Similarly

$$\Pi_{2yij}=\frac{t^*_{xi}}{t_{yij}}\left[\int\limits_{x_i-t_{x,i-1}/2}^{x_i+t_{xi}/2}E_{2y}(x)\delta_2(x)dx\right]^{-1}. \quad (26)$$

Average value of thermal linear expansion coefficients along x-axis at material properties arbitrary variation can be estimated as:

$$\alpha^*_{1xi}=\frac{1}{t_{xi}}\int\limits_{x_i}^{x_i+t_{xi}}\alpha_{1x}(x)dx;\ \alpha^*_{2xi}=\frac{1}{t_{xi}}\int\limits_{x_i}^{x_i+t_{xi}}\alpha_{2x}(x)dx. \quad (27)$$

To define coefficients α^*_{1y}, α^*_{2y} notion of **average** deformation along y-axis was used, so we can obtain following dependencies:

$$\alpha^*_{1yi}=\frac{1}{t^*_{xi}}\int\limits_{x_i-\frac{t_{x,i-1}}{2}}^{x_i+\frac{t_{xi}}{2}}\alpha_{1y}(x)dx;\qquad \alpha^*_{2yi}=\frac{1}{t^*_{xi}}\int\limits_{x_i-\frac{t_{x,i-1}}{2}}^{x_i+\frac{t_{xi}}{2}}\alpha_{2y}(x)dx. \quad (28)$$

Therefore necessary parameters of joining articles are defined. One should note that these parameters are functions on composite articles geometry, physical and mechanical properties and their structure.

4.2 Joining layer compliance

Methods, Assumptions and Procedures

The essence of force connection compliance (like article compliance) is deformation under unit loading ($Q_{xi}=1$, $S_{yi}=1$).

Generally the structure of force connection row is not uniform, that is why equivalent compliance of row consisting of *k* component types can be defined both through compliances of individual components [37] and integration rigidities of force connection components.

To obtain dependencies for practical analysis following assumptions should be established:

- components of joining layer possess same deformation;
- entire load transferring by joint is distributed between force connection components (equilibrium condition).

Condition of deformation compatibility is written as

$$\Delta_i=\Delta_{i1}=\Delta_{i2}=\ldots=\Delta_{ij}=\ldots=\Delta_{ik}, \quad (29)$$

where Δ_i – entire deformation of force connection row;

Δ_{ij} – deformation of *j*-th type component (for example, row of the uniform force points like adhesive, rivets, micro-pins etc.).

Deformations can be estimated as

$$\Delta_i=\frac{Q_{xi}\Pi_{3xzi}}{f_i};\qquad \Delta_{ij}=\frac{Q_{xij}\Pi_{3xzij}}{f_{ij}}, \quad (30)$$

where Q_{xi}, Q_{xij} – shear forces applied to entire *i*-th connection row and individually to *j*-th component in this *i*-th row;

Π_{3xzi}, Π_{3xzij} – correspondent shear compliance of unit area of entire connection row and component of *j*-th type in the plane *xz* (so-called **specific compliance** of connection component);

f_{ij} – area of connection component of *j*-th type in *i*-th row;

f_i – area of *i*-th force connection row

$$f_i = Bt^*_{xi} \;.$$

The assumption about load distribution between connection components gives

$$Q_{xi} = Q_{xi1} + Q_{xi2} + \ldots + Q_{xij} + \ldots + Q_{xik} \,. \tag{31}$$

Using (29), (30) we can express forces transferring by each row force connection by means of force transferring by entire row

$$Q_{xij} = Q_{xi} \frac{\Pi_{3xzi} f_{ij}}{\Pi_{3xzij} f_i}, \tag{32}$$

where $j = 1..k$.

After inserting (32) to (31) we can obtain

$$1 = \frac{\Pi_{3xzi}}{f_i} \sum_{j=1}^{k} \frac{f_{ij}}{\Pi_{3xzij}}. \tag{33}$$

Results and Discussions

Expression (33) permits to define compliance of individual force connection per unit area

$$\Pi_{3xzi} = f_i \cdot \left[\sum_{j=1}^{k} \frac{f_{ij}}{\Pi_{3xzij}} \right]^{-1} . \tag{34}$$

For the case of shear forces the row compliance of combined connections is defined similarly and formula for compliance estimation is

$$\Pi_{3yzi} = f_i \cdot \left[\sum_{j=1}^{k} \frac{f_{ij}}{\Pi_{3yzij}} \right]^{-1} , \tag{35}$$

where Π_{3yzi}, Π_{3yzij} – correspondingly specific shear compliance of entire connection row and *j*-th component type in the plane *yz*.

To define force connection compliance for lateral forces analysis force connection are should be restricted by adopted discreetization spacing in longitudinal and lateral direction

$$f_{il} = t^*_{xi} t_{yil} \,, \tag{36}$$

where t_{yil} – discreetization spacing in longitudinal and lateral direction within *i*-th row at *l*-th section.

The compliance of combined force connection in lateral direction, considering (36), is defined by dependence

$$\Pi_{3yzil} = f_{il} \cdot \left[\sum_{j=1}^{k} \frac{f_{ij}}{\Pi_{3yzilj}} \right]^{-1} . \tag{37}$$

4.3 Determination of joint elements stressed state

Methods, Assumptions and Procedures

Analysis scheme based on joint one-dimensional model gives approximate solution and doesn't

permit ti estimate stress in arbitrary point. It was mentioned earlier that we can operate with **average stress** value at the stage of preliminary design. That is why we can characterize joint stress state with **average stress** in analyzing article sections and in joining layer components.

Average stresses can be estimated by following dependencies, considering above-mentioned assumption about uniform stress distribution within analyzing section:

$$\sigma_{1xi}=\frac{N_{1i}}{\delta_1(x)B};\quad \sigma_{2xi}=\frac{N_{2i}}{\delta_2(x)B};$$
$$\tau_{1xyi}=\frac{q_{1xi}}{\delta_1(x)B};\quad \tau_{2xi}=\frac{q_{2xi}}{\delta_2(x)B};\tag{38}$$
$$\sigma_{1yij}=\frac{N_{1yij}}{\delta_1^*(x)t_{xi}^*};\quad \sigma_{2xi}=\frac{N_{2yij}}{\delta_2^*(x)t_{xi}^*},$$

where $\delta_1(x),\delta_2(x)$ – articles thickness in analyzing section;

$$\delta_1^*(x)=\frac{1}{t_{xi}^*}\int_{x_i-t_{xi}/2}^{x_i+t_{x,i+1}/2}\delta_1(x)dx;\quad \delta_2^*(x)=\frac{1}{t_{xi}^*}\int_{x_i-t_{xi}/2}^{x_i+t_{x,i+1}/2}\delta_2(x)dx.$$

Assuming that joining layer carries shear load only one can calculate **average shear stress** as

$$\tau_{xzi}=\frac{Q_{xi}}{f_i},\tag{39}$$

where f_i – area of force connections in the joining plane.

For combined joining layer we should know both average shear stress and exact stresses in each component

$$\tau_{xzij}=\frac{Q_{xij}}{f_{ij}},\tag{40}$$

where Q_{xij} – force transferring by *j*-th force component;

f_{ij} – area of *j*-th connection component in joining plane.

Results and discussion

Inserting (32) to (40) permits to obtain expression for determination shear stress in each joining layer component

$$\tau_{xzij}=\frac{Q_{xi}}{f_i}\frac{\Pi_{3xzi}}{\Pi_{3xzij}},\tag{41}$$

assuming (39) one can obtain

$$\tau_{xzij}=\tau_{xzi}\frac{\Pi_{3xzi}}{\Pi_{3xzij}}.\tag{42}$$

Therefore we reveal strong dependence between stresses in joining layer components, stipulated by initial assumptions (29), i.e.

$$\tau_{xzi}\Pi_{3xzi}=\tau_{xzi1}\Pi_{3xzi1}=\ldots=const.\tag{43}$$

Value of *const* is defined by the most loaded component [37].

Forces in y-direction due to both external shear loading and lateral deformation restriction appear in joining layer at joint loading with combined (normal and shear) loading. For this case stresses due to shear loading should be combined by vector rule in process of joining layer stressed state analysis

$$\bar{\tau}_{yzij}=\bar{\tau}_{yzi}+\bar{\tau}^*_{yzij},\tag{44}$$

where $\bar{\tau}_{yzi}$ – shear stress in joining layer due to shear force transferring;

$\bar{\tau}^{*}_{yzij}$ – shear stress in joining layer due to normal deformation restriction in lateral direction.

Thus loading intensity in arbitrary section of joining layer and articles will be variable.

Conclusions

1. Suggested analysis method permits to define internal forces in lateral direction at joint loading with normal forces and thermal field influence. Thus unified analysis scheme for joint elements stressed state determination based on physical discreetization approach is synthesized. This analysis scheme naturally considers structure and composition of joining layer and variable character of joining composite articles thickness and their physical and mechanical properties (composite permits to vary both thickness and its physical and mechanical properties).

2. Using proposed approach one can estimate article compliance coefficients, joining layer compliance coefficients and necessary parameters of joining articles, more over these parameters can be functions on joining composite articles geometry, physical and mechanical properties and their structure. Practical engineer dependencies for compliance of combined force connection in lateral direction is derived too.

3. Dependence between stresses in arbitrary section of joining layer components at arbitrary thermal and mechanical loading along joint length is proved and the method for its estimation is done.

Section 5. Review of mathematical methods for metal-composite joint stress-strain analysis

Methods, Assumptions and Procedures

У звітах NASA reports have shown that FEA is suitable for joints stress analysis. This approach has its own advantages: universality by loading and geometry of joining articles; possibility of stress three-dimensional analysis that ensures complete analysis of joint components, possibility of application certified analysis packages like “COSMOS”, “NASTRAN”, “ANSYS” etc, that reduces probability of random errors at FE model composing. FEM disadvantages are: above-mentioned packages are based on virtual displacements principle require significant previous researches for creation super-element near joint with exact boundary conditions; FE selection and definition of its parameters have to be correct; obtained analysis results depends significantly on boundary conditions selection; high computer resources consumption for 3D analysis; practically impossibility of experimental verification of stress value at FE 3D modeling. Application of FEM based on virtual stress (force) principle permits to solve problems of boundary conditions. Above-considered joint model can be related this variant of FE analysis.

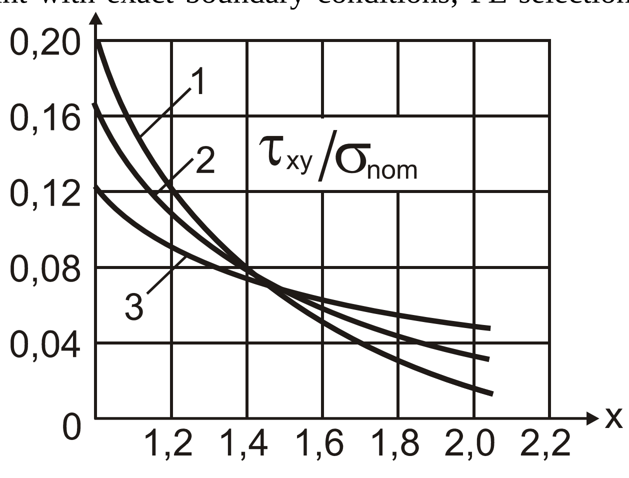


Fig. 19 Comparison different models applied for double-lap adhesive joint analysis (unidirectional plastics ar jointed by epoxy binder): 1- FEM modeling with adhesive layer thickness close to zero; 2- FEM modeling with real thickness adhesive layer; 3- modified Folkersen model; τ_{xy}- shear stress in adhesive layer, σ_{nom}- normal stress in joining articles

Comparing results of FE joint elements stress analysis applied to lap joint and suggested analysis schemes are shown at the fig. 19.

Not less significant is comparison of stress value in adhesive joint joining layer calculated by precise 2D models suggested by Kutinov and disciples (those models are recommended for checking calculations by TsAGI guiding technical materials) with stress values calculated according to suggested model. Analysis of introducing composite articles to load-carrying was conducted (fig. 20) [30].

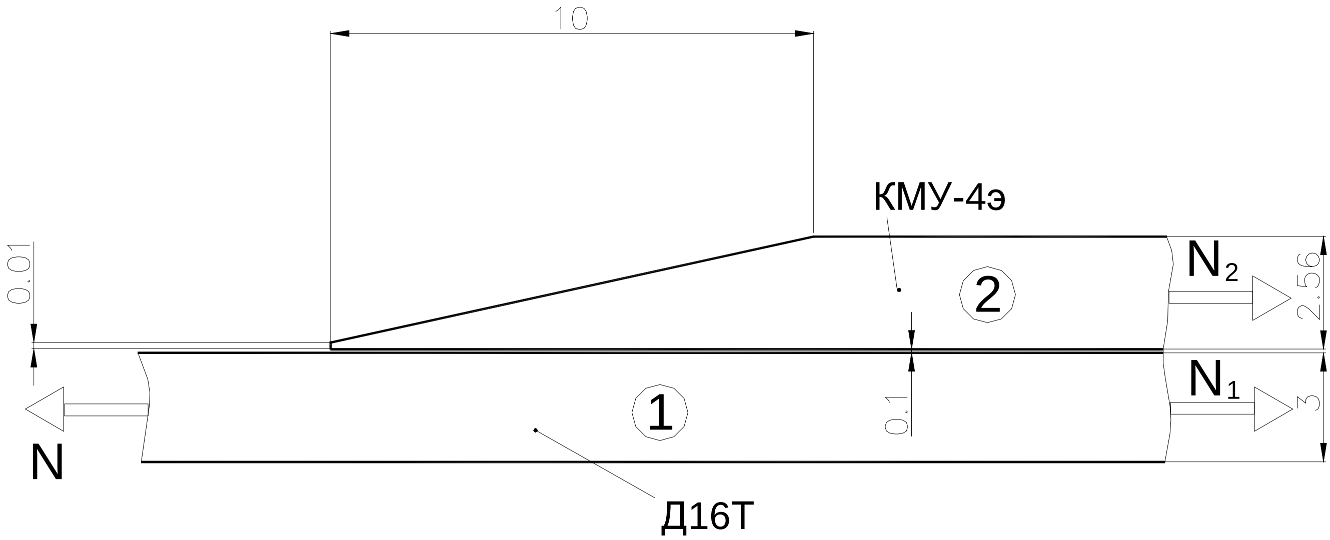


Fig. 20 Case of introducing composite article to load-carrying

Previous researches [25] have shown that stress in adhesive layer is reduced up to zero at the distance 30 mm from joint edge and stress at left joint edge don't effect on stress at right edge. That is why it is enough to consider one of two edges only. Fig. 21 shows stress in adhesive layer calculated by Ionov's analytical model [30] and by developed joint model having different discretization degree. Fig. 22 shows relative stress deviation (calculated by discrete model) from analytical value form different variants of discretization.

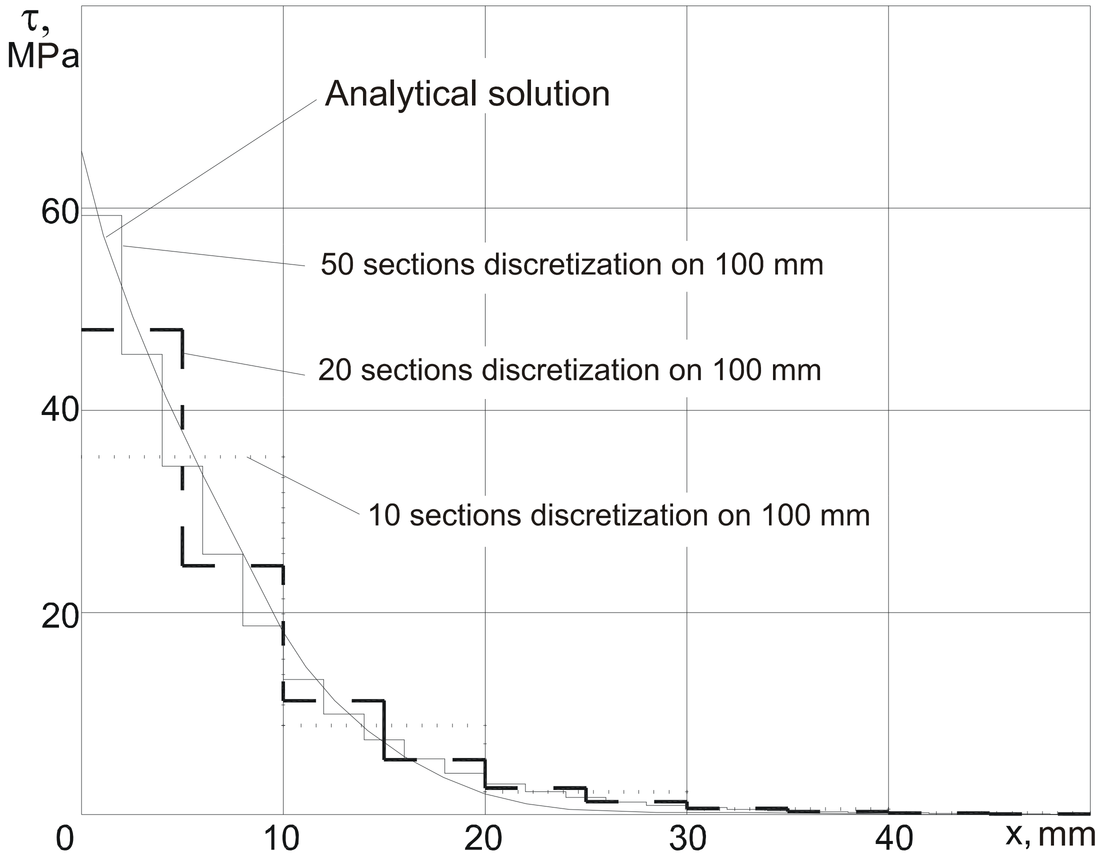


Fig. 21 Stress distribution in adhesive layer near butt edge

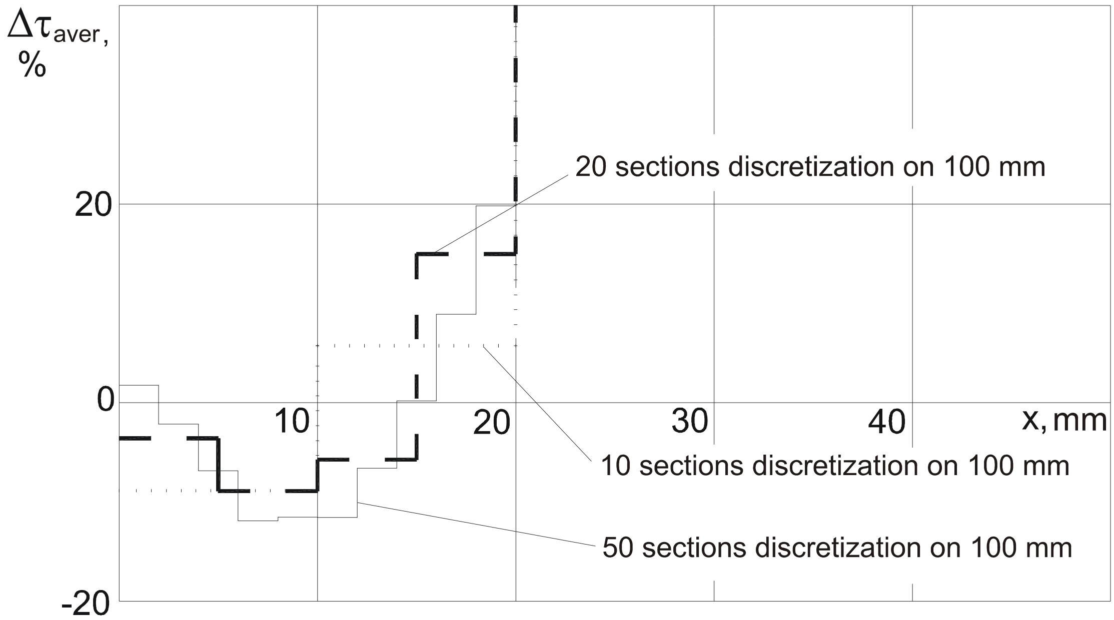


Fig. 22 Relative average stress deviation in adhesive layer from analytical values

It was established that for exceeding enough modeling degree of adhesive layer it is necessary to ensure discretization degree more than 20 sections on 100 mm.

To analyze pure mechanical joints TsAGI guiding technical materials recommend to use model described in [6]. Checking deviation of maximum stress in joining elements from its basic values was conducted at initial data shown in the table 6.

Table 6 – Parameters of join elements

Parameter	Dimension	Variation range
Joint width, B	mm	50
First composite article thickness, δ_1	mm mm mm	2.56 5.12 10.24
Second composite article thickness, δ_2	mm mm mm	3 6 12
Relative fasteners spacing, $\overline{t_x}$ *	——	2, 3, 4, 5, 10
Fastener diameter, d	mm	1, 2, 3, 4, 5, 6, 8, 10
Fasteners quantity in row, m	mm	1, 4
Fastener material	——	30ХГСА, В96

* Is defined as ratio of fasteners installation spacing to its diameter

It was establish previously that inside range of parameters variation maximal stress increase steadily closing to asymptotic value when rows quantity exceeds 15 – 20. Therefore to estimate maximum stress deviation in fasteners stipulated by different models application one can analyze joint consisting of 20 rows.

Results of research are shown at the fig.23 – 26.

Results and Discussion.

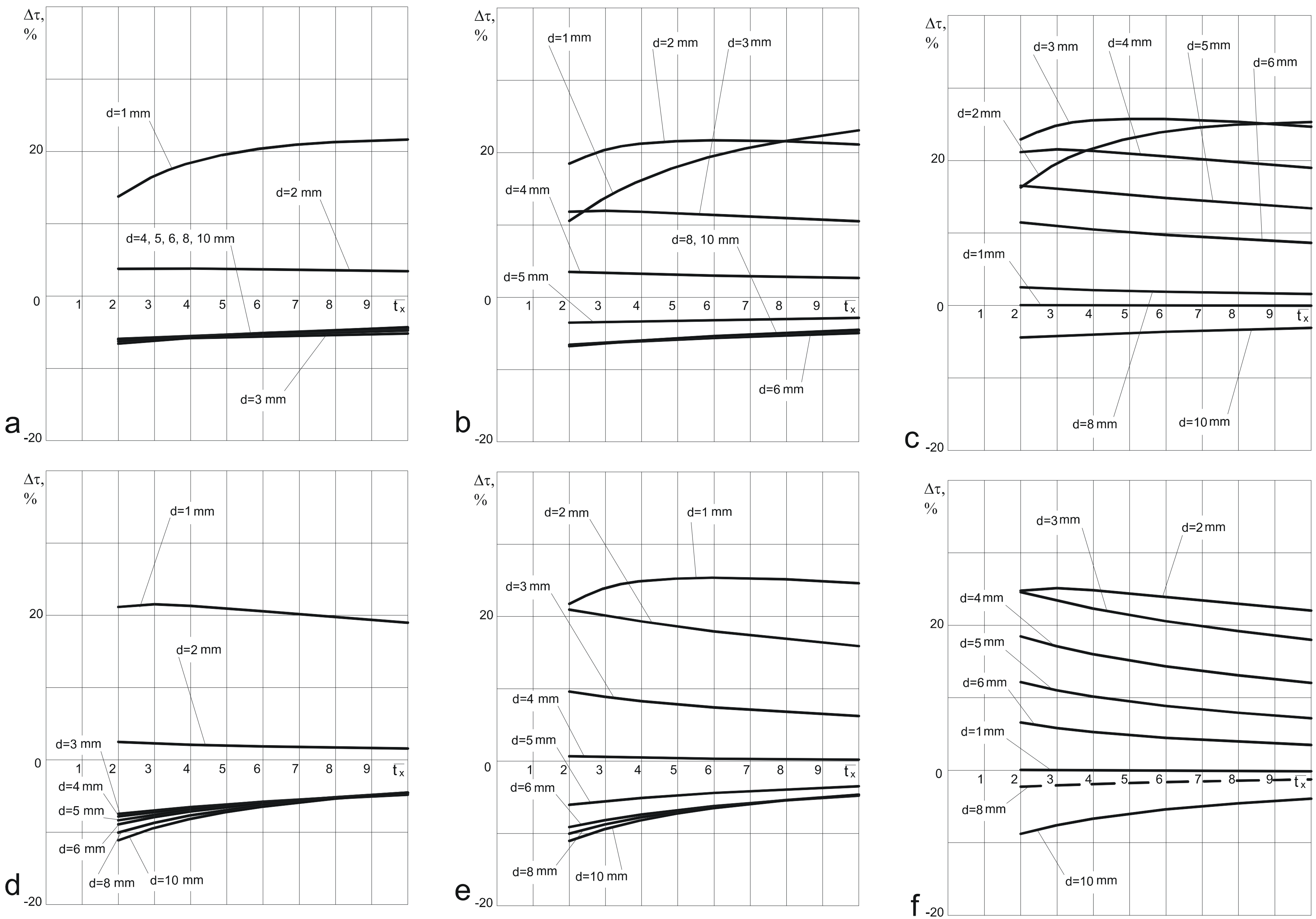


Fig. 23 – Relative deviation of maximum stress at using Boeing model (fasteners material is 30ХГСА)

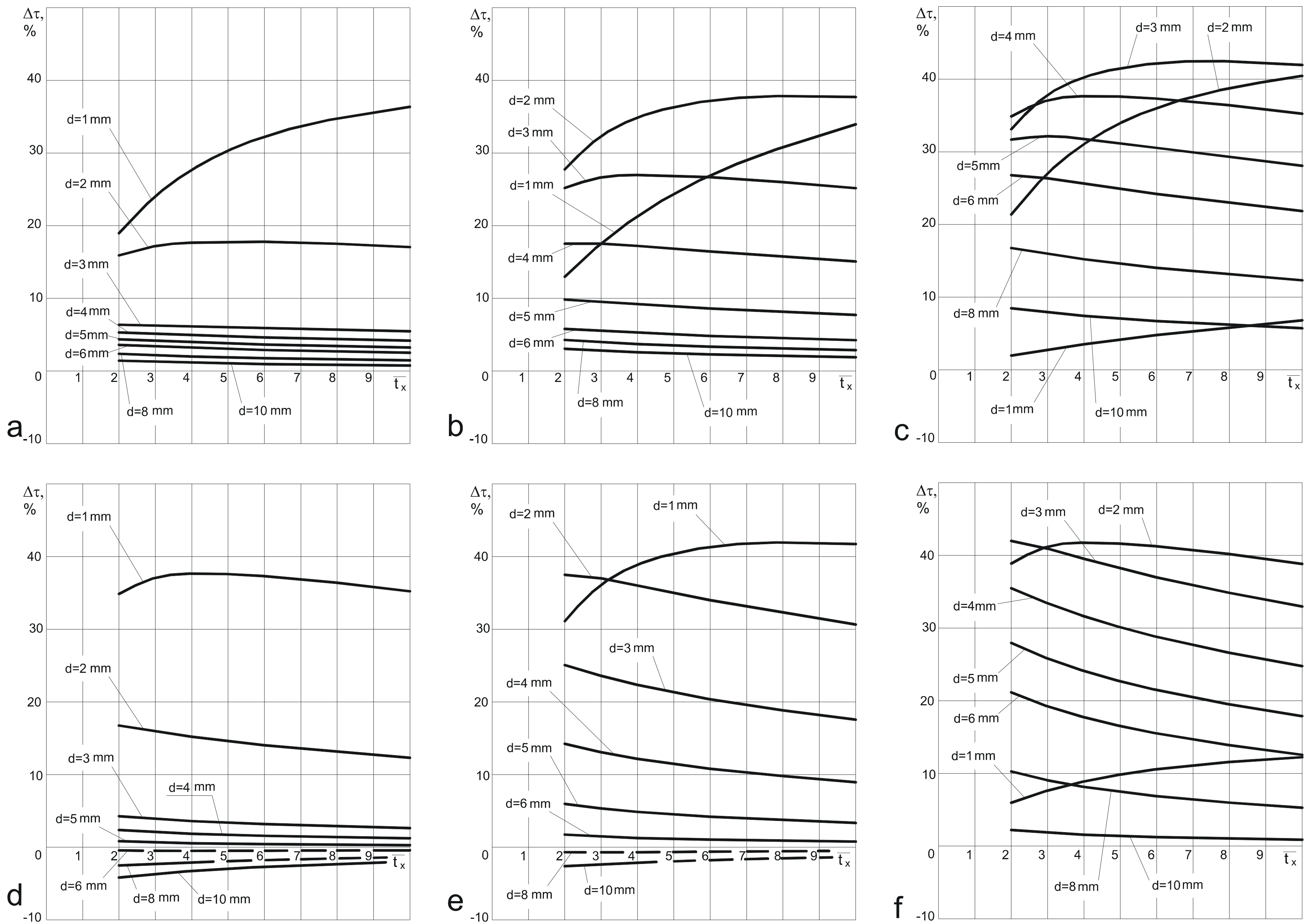


Fig. 24 – Relative deviation of maximum stress at using Boeing model (fasteners material is B96)

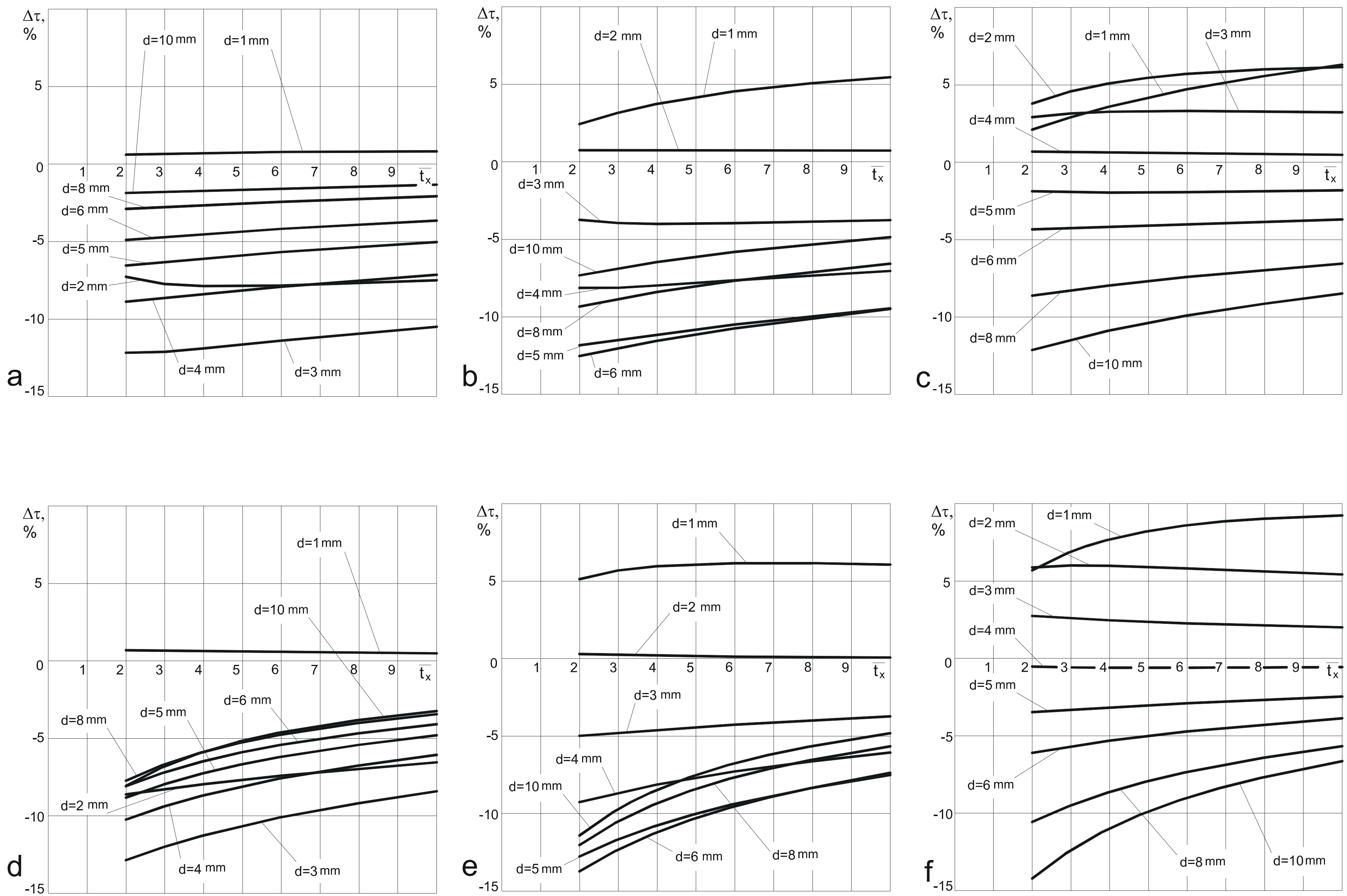


Fig. 25 – Relative deviation of maximum stress at using Douglas model (fasteners material is 30ХГСА)

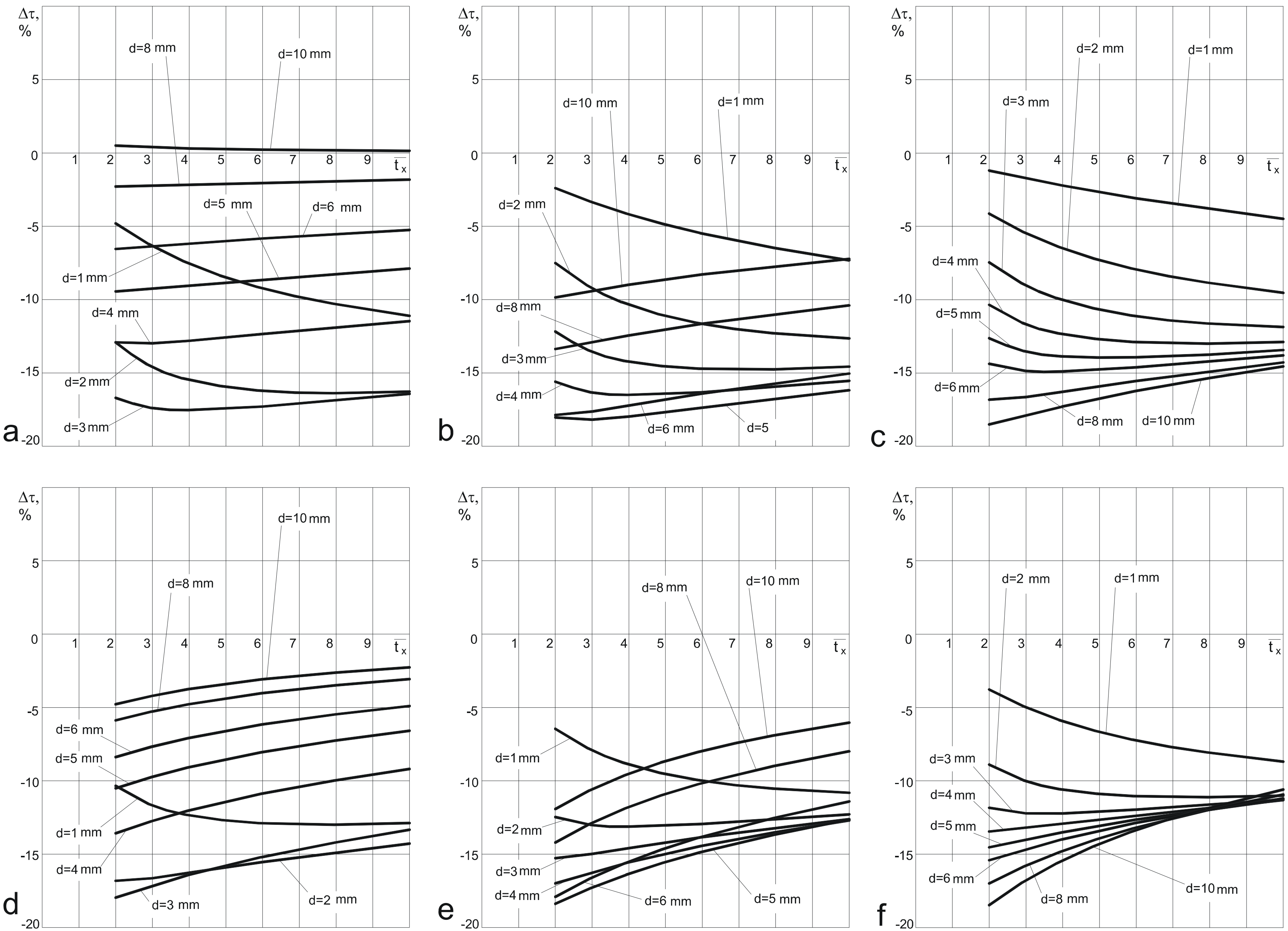


Fig. 26 – Relative deviation of maximum stress at using Douglas model (fasteners material is B96)

Fig. 23 – 26 contain following information: (a) – (c) – for case of single fastener in the row; (d) – (f) – for case of four fasteners in the row; (a) and (d) – for articles with initial thickness ($\delta_1 = 2.56$ mm and $\delta_1 = 3$ mm); (b) and (e) – for articles with double thickness ($\delta_1 = 5.12$ mm and $\delta_1 = 6$ mm); (c) and (f) – for articles with quadruple thickness ($\delta_1 = 10.24$ mm and $\delta_1 = 12$ mm).

Conclusions

Following conclusions can be done after analysis of above-mentioned figures:

– maximum stress in fastener depends very slightly on fastener relative cross-section (ν_{fastener});

– stress value deviation from reference value depends on fasteners installation spacing very slightly;

– influence of joining articles thickness increasing on dependence of maximum stress deviation on fasteners installation spacing in its turn depends on used fastener model. Thus using Boeing model at joining articles thickness increasing causes increasing influence degree of fastener diameter on relative stress deviation in fastener. For Douglas model character of influence generally depends on fastener material. Thus fastener made of high modulus material (30ХГСА) at increasing thickness of joining articles causes growing influence degree of fastener diameter on relative stress deviation, but application low-modulus materials for fasteners manufacturing (В96) causes mentioned influence degree to be pure;

– each model has its own application field: Boeing model can be recommended for cases fastener aspect ratio doesn't exceed two, in other cases auxiliary research is needed; Douglas model gives quite good convergence of analysis results (so steady reduction of maximum stress can be observed), that is why application field of this model depends on allowed range of stress deviation.

Section 6. Studying influence of compliance on joint stress state

Methods, Assumptions and Procedures

Following conclusion can be done from previous section: joint elements compliance defines mainly these components stress-strain state. It was established that estimation of joining articles compliances can be conducted by well grounded models that describes well physical process of load transfer. At the same time joint elements compliance can be realized using different models developed by design bureaus and organizations. Obtained numerical values of joint elements compliances differ significantly from each other and from experimental values. Previous researches of joining layer compliance variation influence on joint load carrying ability have revealed its non-linear character. Therefore this is a necessity of determination confidence interval for compliance at which maximum stress in joining layer varies inside predefined range.

This research was conducted by two stages: first stage – analysis of adhesive joint based on analytical model; second stage – studying mechanical joint based on discrete one-dimensional model. To simplify analysis the case of load transferring between articles having the same rigidity and without temperature field influence was considered.

6.1. Adhesive joint analysis

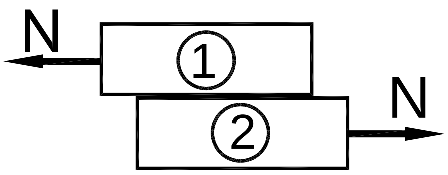


Fig. 27 – Adhesive joint model

According to used assumptions stress distribution in adhesive layer of considered joint (fig. 27) is described by following dependence

$$\tau(x) = C_1 \operatorname{sh} kx + C_2 \operatorname{ch} kx \;, \tag{45}$$

where $C_1 = A$; $C_2 = \dfrac{B - A \operatorname{ch} kL}{\operatorname{sh} kL}$;

$$A = N\left(\frac{\Pi_{2x}}{k\Pi_c} - k\right);\ B = N\frac{\Pi_{2x}}{k\Pi_c};\ k = \sqrt{\frac{\Pi_{1x} + \Pi_{2x}}{\Pi_c}};$$

Пс – adhesive layer compliance.

Stress pikes in adhesive layer occur at joint edges:

at $x = 0$ $$\tau(0) = C_2 = \frac{B - A\,\mathrm{ch}\,kL}{\mathrm{sh}\,kL}; \quad (46)$$

at $x = L$ $$\tau(L) = \frac{B\,\mathrm{ch}\,kL - A}{\mathrm{sh}\,kL}. \quad (47)$$

Therefore at constant adhesive strength joint load-carrying ability depends on compliance of joining articles, adhesive layer and joint length. Increasing joint length causes maximum stress reduction, but at $L \geq L_{lim}$ [29] stress level becomes stable and equal to

$$\tau(0) = -A \text{ та } \tau(L) = B. \quad (48)$$

This fact to exclude joint length as structural parameter influencing on joint load-carrying ability. Ti simplify analysis it was solved to derive articles compliance with adhesive layer compliance

$$f = \frac{\Pi_c}{\Pi_{1x}}. \quad (49)$$

In this case boundary values of maximum stress in adhesive layer (48) (considering assumption $\Pi_{1x} = \Pi_{2x}$) is defined by formula

$$\tau_{max} = \tau(0) = \tau(L) = \frac{N}{\sqrt{2f}}. \quad (50)$$

Exactly this dependence was used for governing equation of maximum stress as function of adhesive layer compliance. It is possible to distinguish two variants of this dependence (related to each other):

– at compliance reduction $f' = f - \xi_1$ maximum stress will increase $\tau'_{max} = \tau_{max} + \xi$, so for this case

$$\bar{\xi}_1 = \frac{2\bar{\xi} + \bar{\xi}^2}{\left(1+\bar{\xi}\right)^2}; \quad (51)$$

– at compliance increasing $f' = f + \xi_1$ maximum stress will reduce $\tau'_{max} = \tau_{max} - \xi$, so for this case

$$\bar{\xi}_1 = \frac{2\bar{\xi} - \bar{\xi}^2}{\left(1-\bar{\xi}\right)^2}. \quad (52)$$

Here $\bar{\xi}_1$ – allowable relative deviation (relative estimation error) of adhesive layer compliance; $\bar{\xi}$ – predefined relative deviation of maximum stress in adhesive layer.

For predefined value of stress deviation $\bar{\xi} = \pm 5\%$ allowable relative deviation of compliance is $\bar{\xi}_1 = -9.3\ldots+10.8\%$. It is necessary to note that relationship between stress deviation and compliance deviation doesn't depend on geometrical and mechanical parameters of adhesive joint components.

6.2. Mechanical joint studying

Results obtained during adhesive joint analysis permitted to hope that dependence between maximum stress deviation and relative error of joint elements compliance don't depend on joining method. To confirm this assumption it was necessary to study mechanical joint behavior. But available model of mechanical joint is discrete one that is why one has to change approach to research formulation comparing with adhesive joint.

The main idea of research is estimation joint load-carrying ability at predefined conditions, i.e. parameters of joining articles, fasteners row quantity, fastener material properties and selected analytical model of force connection. Then it is possible to freeze constant values of defined joint parameters and level of joint loading (for example, by means of stress in regular zone of joining articles) and conduct variation of fastener compliance value up to those magnitudes at which maximum stress inside fastener will differ from reference (initial) values on predefined value. For example, in this research fastener ultimate shear strength was selected as reference value and range of this value variation is selected to be equal ±5%. Thus dependence between allowable range fastener compliance on joint parameters is obtained.

One should know that absolute values of fasteners parameters lay inside range $10^{-5}\ldots10^{-8}$ mm/N, that is why normalizing fastener compliance refer to joining articles compliance have been done to simplify research procedure and results analysis.

To study pure mechanical joint (joining articles have the same parameters) following data were established (see table 7).

Table 7 – Main parameters of joint used for parametrical analysis

Joining articles parameters	Measuring units	Value
Joining articles thickness δ_{art}	mm	5.0; 10.0
Article elasticity modulus $E_{Д}$	GPa	50; 100; 150; 200; 250
Micro-fastener diameter $d_{fastener}$	mm	1.0; 6.0; 10.0
Micro-fastener elasticity modulus $E_{fastener}$	GPa	70; 100; 200

Parametrical research was conducted according to following algorithm:

1. System of joint parameters (shown in table 5) is frozen. Fastener shear strength is defined (in this studying it is assumed to be equal 600 MPa). Minimal fastener rows quantity is frozen to be equal 3. Arrangement scheme of fasteners installation is 3x3 diameters.

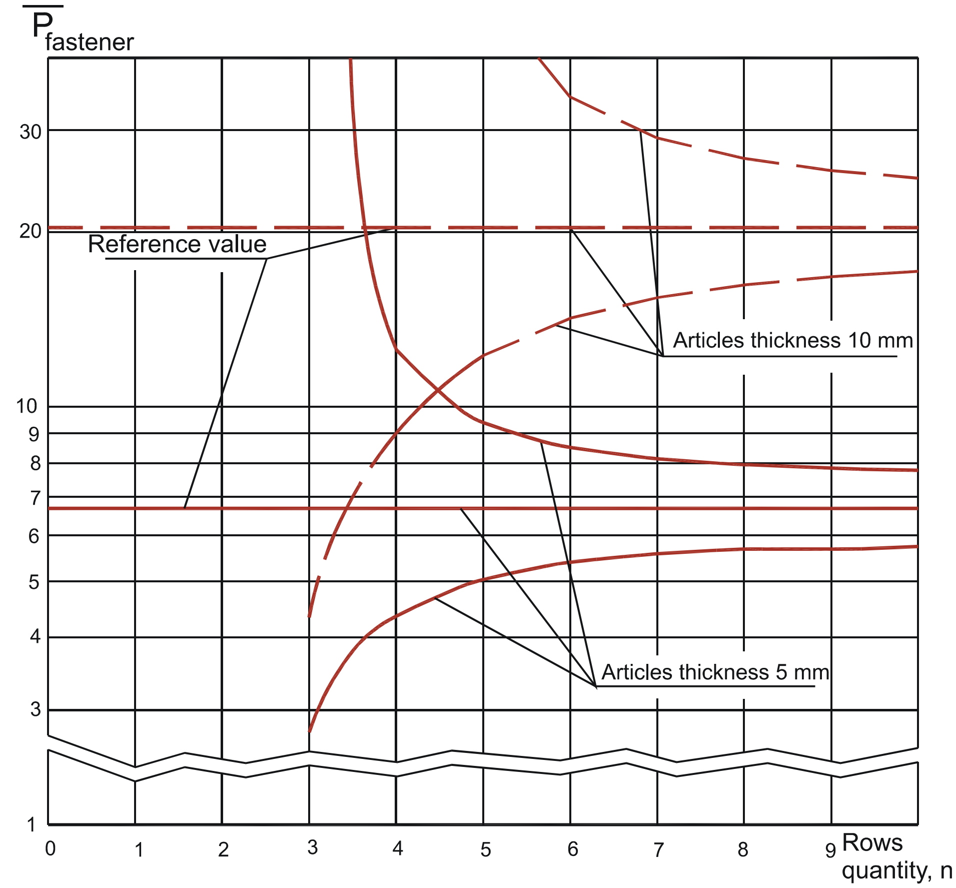
2. Stress in joining articles from external loading is selected in such way to ensure shear stress in most loaded fasteners to be equal fastener material shear strength. For parametrical analysis allowable deviation from reference value ≤+0,1‰ that permits to draw quite smooth curve. Obtained stress in articles is frozen as reference one (it means that joint load-carrying ability is frozen too).

3. Varying fastener compliance we can catch its two margin boundaries which correspond to exceeding (decreasing) shear stress value of the most loaded fastener refer to material shear strength not more then 5%. Value of 5% is allowable stress variation at static loading.

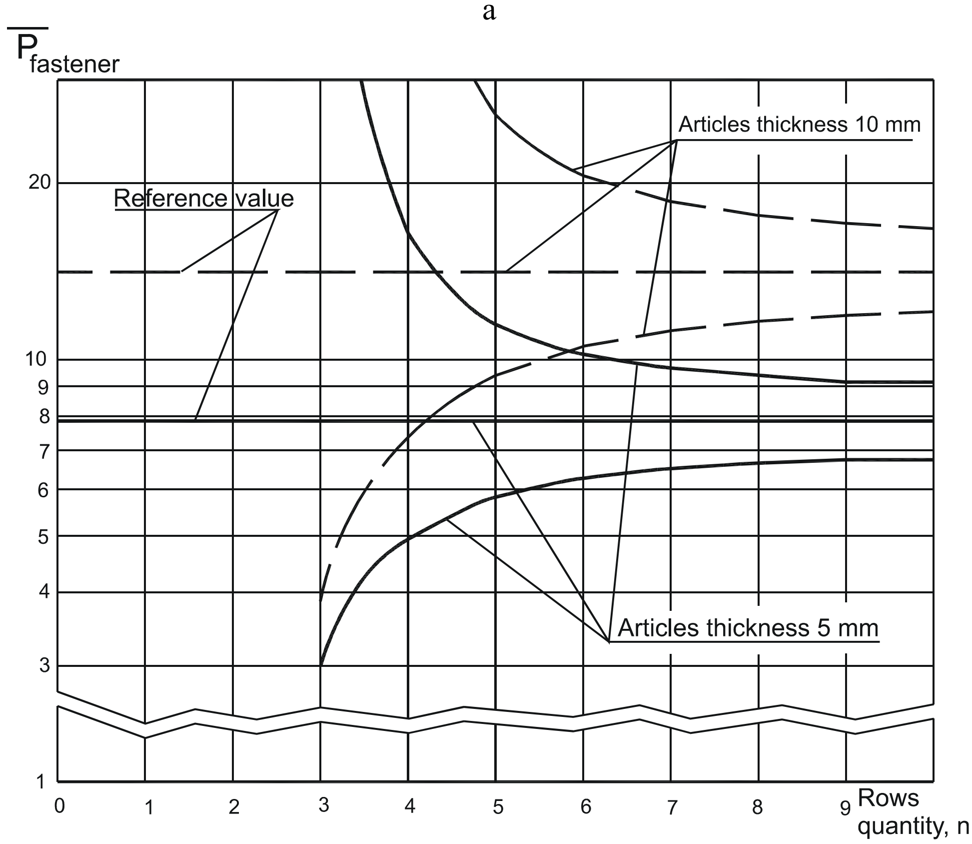
4. If quantity of fastener rows is less than 10 we have to increase it by one and repeat actions mentioned in clause 2 and 3. If quantity of fastener rows is more than 10 we have to change system of initial joint parameters (shown in table 5) and fulfill clauses 1-4.

Results of parametrical researches are shown on fig.28-30 below.

Results and Discussion.

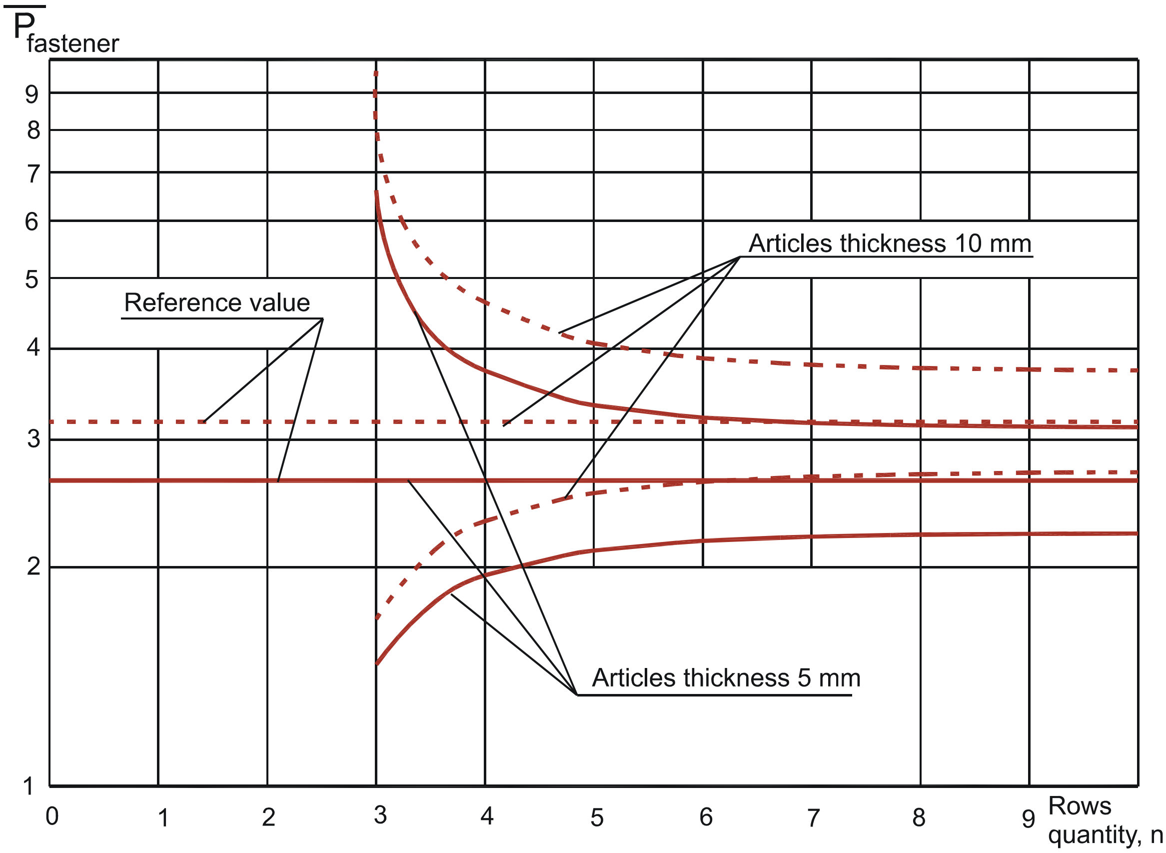


a

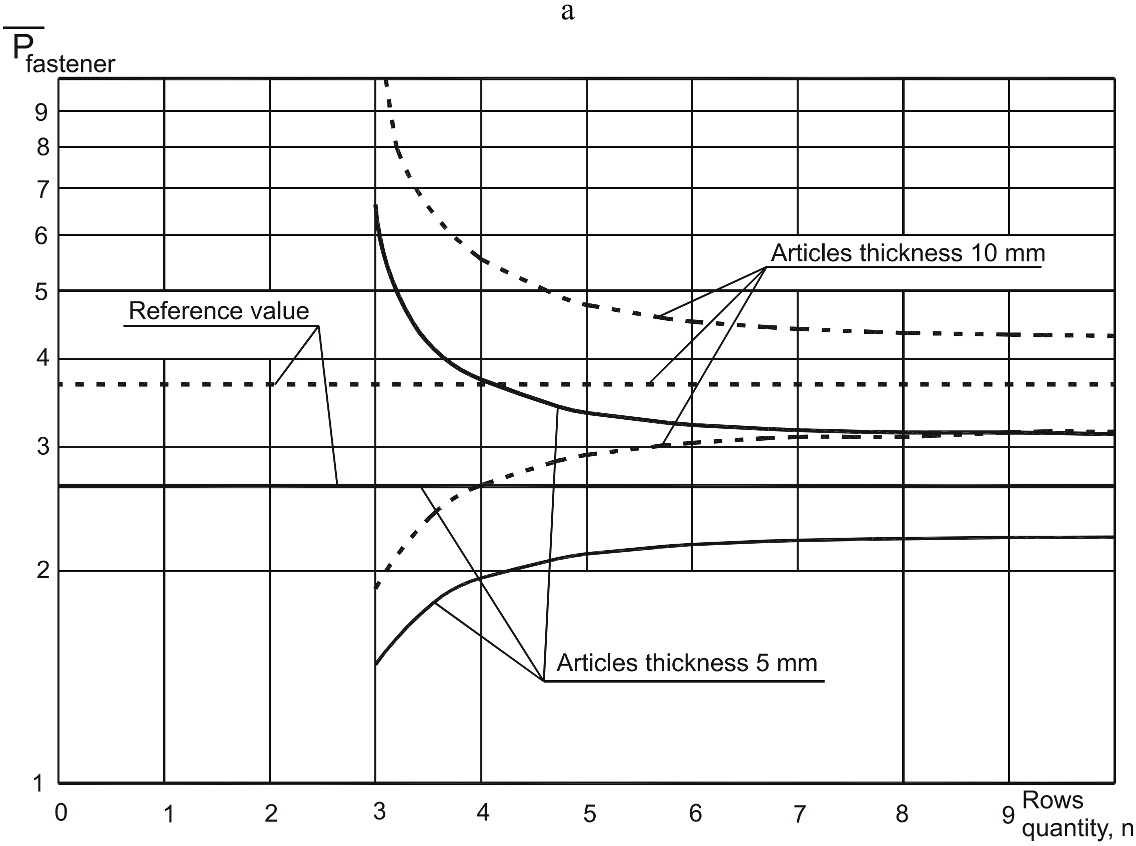


b

Fig. 28 Distribution of boundary zones of fastener relative compliance as function on fastener rows quantity (E_{art}=50 GPa, $E_{fastener}$=200 GPa, $d_{fastener}$=1 mm): a– reference value of fastener compliance is defined by Boeing formula; b– reference value of fastener compliance is defined by Douglas formula

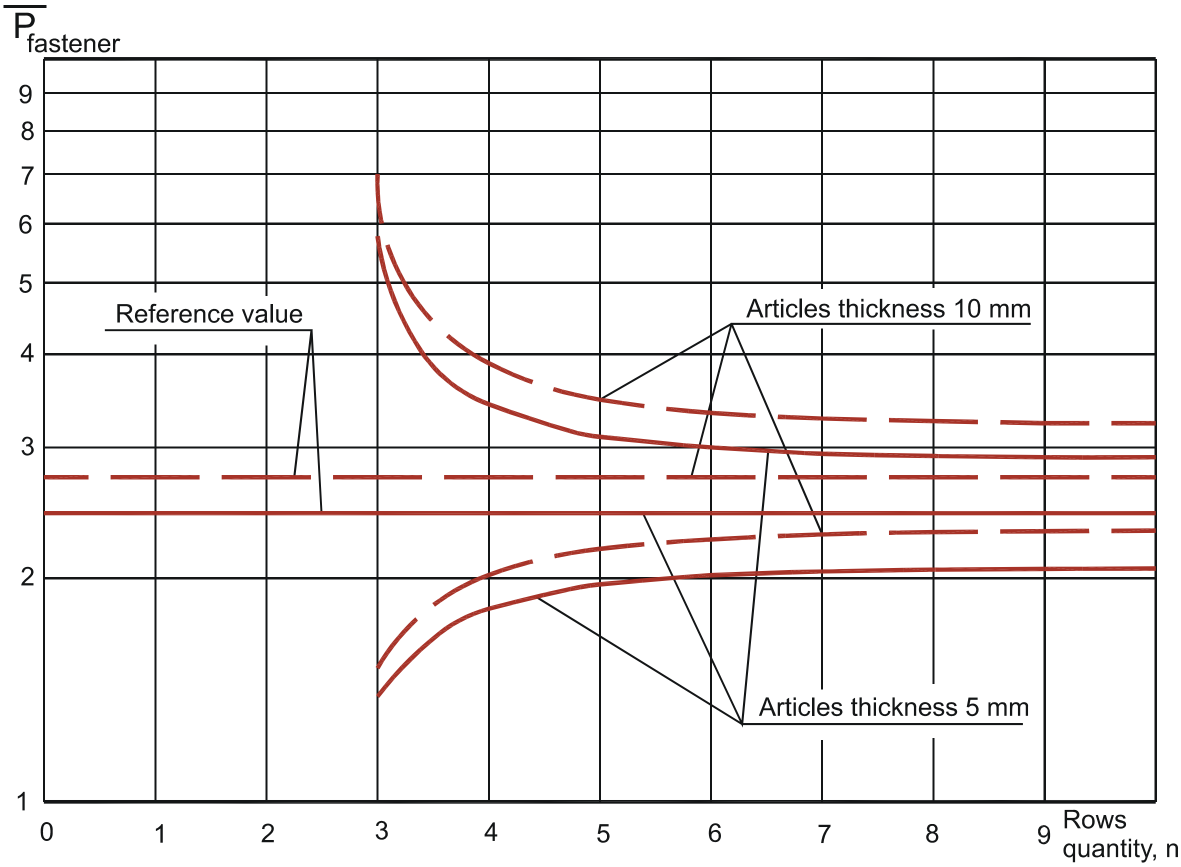


a

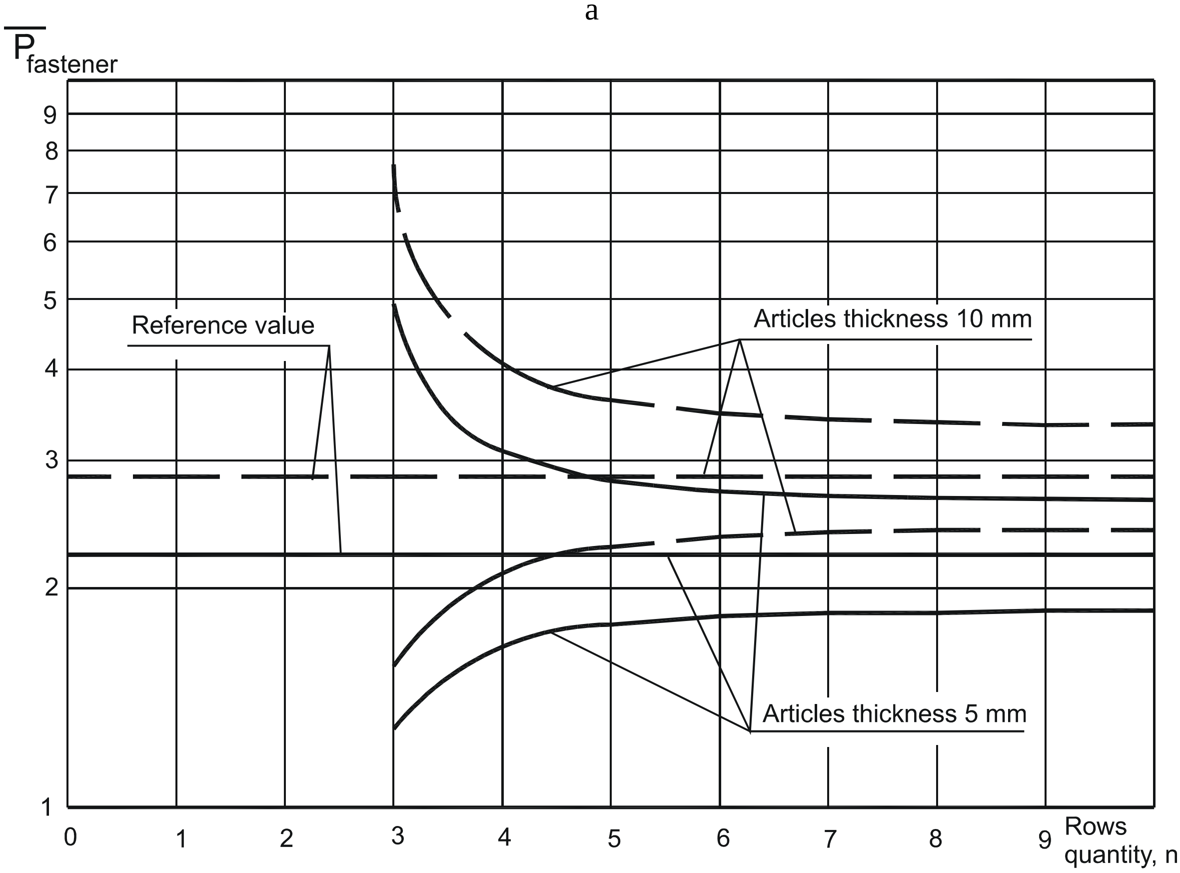


b

Fig. 29 Distribution of boundary zones of fastener relative compliance as function on fastener rows quantity (E_{art}=50 GPa, $E_{fastener}$=200 GPa, $d_{fastener}$=6 mm): a– reference value of fastener compliance is defined by Boeing formula; b– reference value of fastener compliance is defined by Douglas formula



a



b

Fig. 30 Distribution of boundary zones of fastener relative compliance as function on fastener rows quantity (E_{art}=50 GPa, $E_{fastener}$=200 GPa, $d_{fastener}$=10 mm): a– reference value of fastener compliance is defined by Boeing formula; b– reference value of fastener compliance is defined by Douglas formula

It was established that non-uniformity of force distribution along joint length influences on allowable range width of fastener compliance. Allowable relative deviation of fasteners compliance belongs to range $(-9.3\ldots+10.8\%)\Pi_{fastener}$ at exceeding margin load-carrying ability of mechanical joint. This result coincides well adhesive joint analysis. Moreover it was established in the case of application several fastener rows (from 3 to 6) it is more profitable to use fastener models which give overrated compliance value. In specific cases one can use linear dependence of stress distribution between fastener rows, that corresponds to $\Pi_{fastener} \to \infty$ (fig. 31).

Conclusions

1. In the case of margin join load-carrying ability allowable range of joining layer compliance is $-9.3\ldots+10.8\%$ of reference value not-depending on selected joint type.

2. Main complex parameter influencing on fastener compliance confidence interval width are values of its relative compliance.

3. Conditions permitting to consider fasteners compliance to be infinitely large are defined (linear loads distribution along joint). In this case maximum fasteners stress will be reduced within range ±5% (fig. 23).

4. Character of joint elements parameters influence on relative fastener compliance is found. These parameters are fastener diameter, fasteners material modulus, joining articles modulus, joining articles thickness. Increasing fastener diameter or fasteners material modulus cause its relative compliance reduction. Increasing joining articles thickness or its modulus (i.e. increasing joining articles rigidity) causes relative fastener compliance increasing. Joining of compliable articles with rigid fasteners requires more thorough studying of this fasteners compliance.

5. Used at analysis formulas of Boeing and Douglas gave similar results (within range of predefined stress variation). Moreover one can see that at analysis of quite rigid fasteners ($d_{fastener} \geq 6$ mm) relative fastener compliance can be expressed by mean of coefficient of relative compliance escaping of joining articles thicknesses. In this case fastener compliance can be described by quite simple dependence

$$P_{fastener} = \frac{k}{E_{art}\delta_{art}}, \tag{53}$$

where k=2.7…3.2 – coefficient of relative compliance; less value corresponds to fastener diameter $d_{fastener}$=10 mm, largest value corresponds to $d_{fastener}$=6 mm.

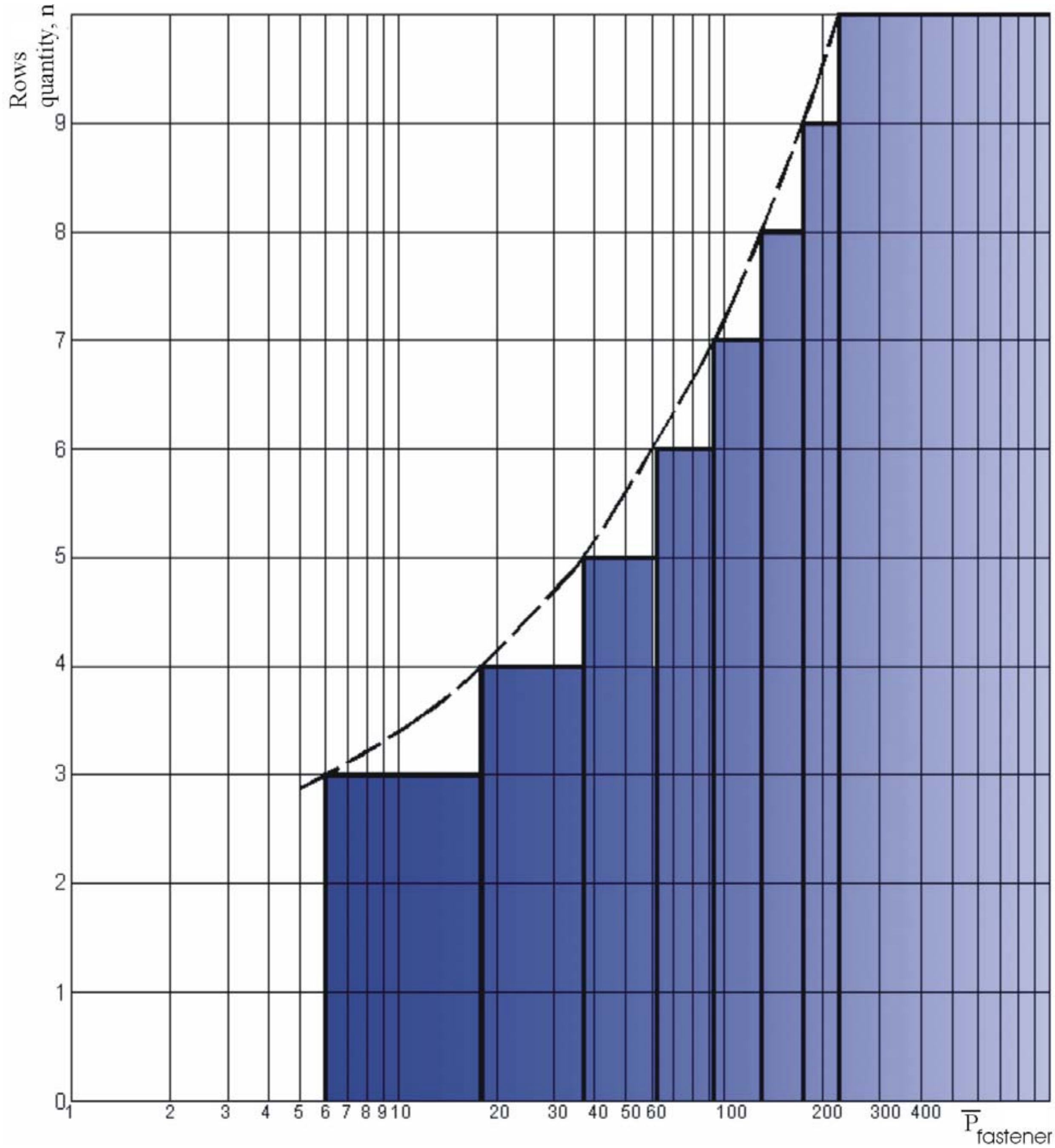


Fig. 31 Dependence between fastener relative compliance and fastener rows quantity which ensures uniform force distribution along joint length at decreasing stress inside fastener (refer to its maximum strength) within 5% (colored area)

Section 7. Synthesis of analytical dependencies for compliance coefficients of micro-fasteners embedded to composite article

Methods, Assumptions, and Procedures

Analysis of fastener variants has shown that fasteners end planed to be embedded to composite article can has a large variety of geometry (fig. 32). Fastener another end can be absent at all if designer suggests to mill or weld it to correspondent metal article.

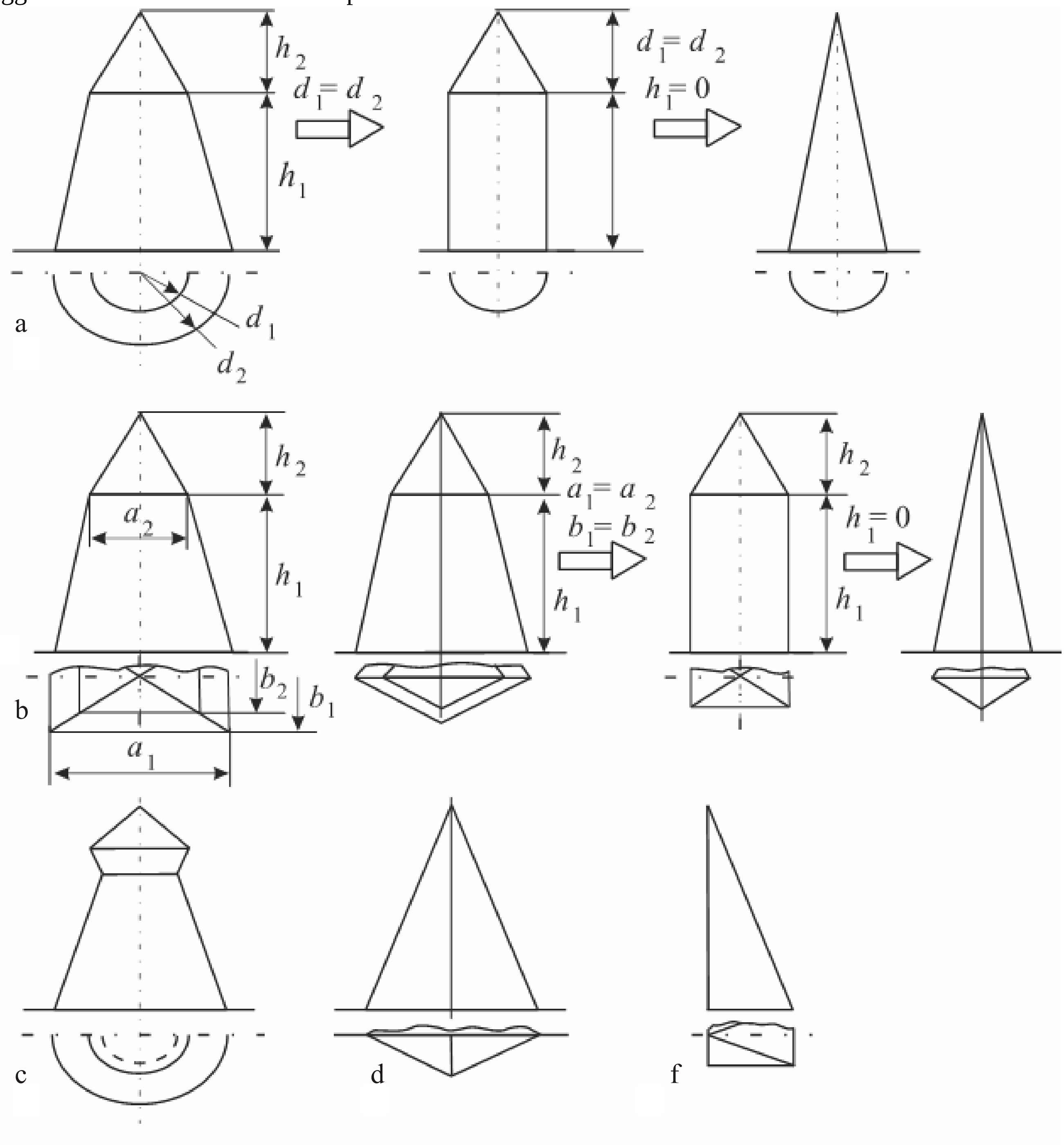


Fig. 32 Possible shape variants of micro-fasteners for embedding to composite article

Conducted analysis of application micro-fastener Douglas and Boeing models for determination of micro-fastener compliance has shown that mentioned models can be used only for cylindrical micro-fasteners which are embedded to joining article through their entire thickness. For other possible cases of micro-fasteners shape (see fig. 32) these models give results having pure validity. Potentially it is possible to replace real micro-fastener with dummy one having the same compliance of force connection

but reasonability of this approach is quite doubtful.

Analytical model (based on Volkersen hypothesis about reduced joining layer) for determination micro-fasteners compliance was suggested:

$$\Pi_{3xi} = (\Pi_{1xzi} + \Pi_{2xzi} + \Pi_{adh\,xzi}) / t^*_{xi}B;$$
$$\Pi_{3yi} = (\Pi_{1yzi} + \Pi_{2yzi} + \Pi_{adh\,yzi}) / t^*_{xi}B; \quad (54)$$
$$\Pi_{3yij} = (\Pi_{1yzi} + \Pi_{2yzi} + \Pi_{adh\,yzi}) / t^*_{xi}t^*_{yij},$$

where $\Pi_{1xzi}, \Pi_{2xzi}, \Pi_{adh\,xzi}, \Pi_{1yzi}, \Pi_{2yzi}, \Pi_{adh\,yzi}$ can be estimated as following:

$$\Pi_{1xzi} = \int_0^{\delta^*_{1i}/2} \frac{dz}{G^*_{1xzi}(z)}; \quad \Pi_{1yzi} = \int_0^{\delta^*_{1i}/2} \frac{dz}{G^*_{1yzi}(z)};$$
$$\Pi_{2xzi} = \int_0^{\delta^*_{2i}/2} \frac{dz}{G^*_{2xzi}(z)}; \quad \Pi_{2yzi} = \int_0^{\delta^*_{2i}/2} \frac{dz}{G^*_{2yzi}(z)}; \quad (55)$$
$$\Pi_{adh\,xzi} = \int_0^{\delta^*_{adh\,i}} \frac{dz}{G^*_{adh\,xzi}(z)}; \quad \Pi_{adh\,yzi} = \int_0^{\delta^*_{adh\,i}} \frac{dz}{G^*_{adh\,yzi}(z)}.$$

In above-mentioned expressions $G^*_{1xz}(z), G^*_{2xz}(z), G^*_{adh\,xz}(z), G^*_{1yz}(z), G^*_{2yz}(z), G^*_{adh\,yz}(z)$ – are reduced moduli of interlaminar shear for first, second articles and adhesive layer in axes XZ and YZ correspondingly. Reduction is conducted by rule of mixture

$$G(z) = G_{fast}\nu_{fast}(z) + G_{mm}(1 - \nu_{fast}(z)), \quad (56)$$

where $\nu_{fast}(z) = f_{fast}(z) / t^*_xB$; $f_{fast}(z)$ – current cross-section area of micro-fastener; G_{fast}, G_{mm} – shear moduli of fastener material and main material (joining articles or adhesive);

$\delta^*_{1i}, \delta^*_{2i}, \delta^*_{adh\,i}$ – average thickness of first, second articles and adhesive layer at the section t^*_{xi}.

Results and Discussion

Specific dependencies for separate cases of micro-fastener shape were derived based on general dependences analysis (table. 8).

Verification of suggested model was done at the same conditions that were used at models of Boeing та Douglas (see table 6). Obtained results are shown at fig. 33, 34. These figures use the same description which was used at fig. 23 – 26 (see comments for them).

Following conclusions were established:

– deviation of maximum value of stress in fastener from reference value depends significantly on arial density of micro-fasteners installation KE (ν_{fast});

– increasing thickness of joining articles causes strong tendency to reduction degree of maximum stress deviation.

Underestimation of micro-fastener compliance and consequent overestimation of maximum stress were clearly observed earlier in KhAI practical tests [29], [39].

Conclusions

1. Model of micro-fasteners permitting to take into consideration exact fastener cross-section, arial density if installation, its side shape and shear modulus in vertical plane is worked out.;
2. Mentioned model was verified by means of comparison with data obtained theoretically in

TsAGI and experimentally in KhAI. Quite high degree of fastener stress over estimation is found that can be explained by under estimation of fastener compliance out of confidence interval. As result application field of suggested model becomes narrower that can be;

3. Special investigation devoted to increasing analysis precision was conducted in KhAI (for example, see Loktionov papers). This investigation has to be prolonged in future.

Table 8 – Compliance coefficients of lateral micro-fasteners row

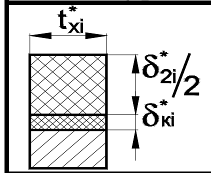
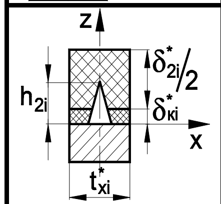
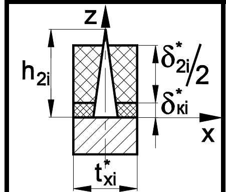
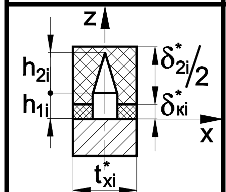
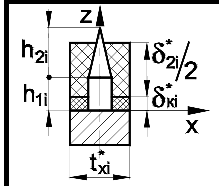
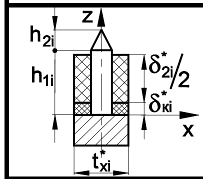
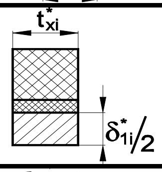
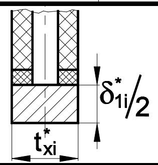
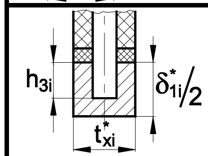
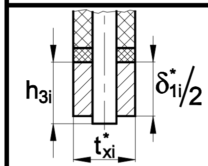
Force connection type	Parameters value	Analytical dependences
	$f_{0i}=0$	$\Pi_{2xzi}=\dfrac{\delta^*_{2i}}{2G_{2xzi}}$; $\Pi_{2yzi}=\dfrac{\delta^*_{2i}}{2G_{2yzi}}$; $\Pi_{adh\,xzi}=\Pi_{adh\,yzi}=\dfrac{\delta^*_{adh\,i}}{G_{adh\,i}}$
	$f_{0i}\neq 0$ $h_{1i}=0$ $h_{2i}\le\dfrac{\delta^*_{2i}}{2}$ $G_{fast\,i}>G_{adh\,i}$ $G_{fast\,i}>G_{2xzi}$ $G_{fast\,i}>G_{2yzi}$	$\Pi_{adh\,xzi}=\Pi_{adh\,yzi}=\dfrac{h_{2i}}{\sqrt{\nu_{0i}G_{adh\,i}\left(G_{fast\,i}-G_{adh\,i}\right)}}\left[\operatorname{arctg}\sqrt{\dfrac{\nu_{0i}\left(G_{fast\,i}-G_{\kappa i}\right)}{G_{adh\,i}}}-\operatorname{arctg}\left(1-\dfrac{\delta^*_{adh\,i}}{h_{2i}}\right)\sqrt{\dfrac{\nu_{0i}\left(G_{fast\,i}-G_{adh\,i}\right)}{G_{adh\,i}}}\right]$; $\Pi_{2xzi}=\dfrac{h_{2i}}{\sqrt{\nu_{0i}G_{2xzi}\left(G_{adh\,i}-G_{2xzi}\right)}}\operatorname{arctg}\left(1-\dfrac{\delta^*_{adh\,i}}{h_{2i}}\right)\sqrt{\dfrac{\nu_{0i}\left(G_{fast\,i}-G_{2xzi}\right)}{G_{2xzi}}}+\dfrac{\delta^*_{2i}-2\left(h_{2i}-\delta^*_{adh\,i}\right)}{2G_{2xzi}}$; $\Pi_{2yzi}=\dfrac{h_{2i}}{\sqrt{\nu_{0i}G_{2yzi}\left(G_{adh\,i}-G_{2yzi}\right)}}\operatorname{arctg}\left(1-\dfrac{\delta^*_{adh\,i}}{h_{2i}}\right)\sqrt{\dfrac{\nu_{0i}\left(G_{fast\,i}-G_{2yzi}\right)}{G_{2yzi}}}+\dfrac{\delta^*_{2i}-2\left(h_{2i}-\delta^*_{adh\,i}\right)}{2G_{2yzi}}$
	$f_{0i}\neq 0$ $h_{1i}=0$ $h_{2i}>\dfrac{\delta^*_{2i}}{2}$	$\Pi_{adh\,xzi}$, $\Pi_{adh\,yzi}$ – see previous scheme $\Pi_{2xzi}=\dfrac{h_{2i}}{\sqrt{\nu_{0i}G_{2xzi}\left(G_{fast\,i}-G_{2xzi}\right)}}\left[\operatorname{arctg}\left(1-\dfrac{\delta^*_{adh\,i}}{h_{2i}}\right)\sqrt{\dfrac{\nu_{0i}\left(G_{fast\,i}-G_{2xzi}\right)}{G_{2xzi}}}-\operatorname{arctg}\left(1-\dfrac{\delta^*_{2i}+2\delta^*_{adh\,i}}{2h_{2i}}\right)\sqrt{\dfrac{\nu_{0i}\left(G_{fast\,i}-G_{2xzi}\right)}{G_{2xzi}}}\right]$; $\Pi_{2yzi}=\dfrac{h_{2i}}{\sqrt{\nu_{0i}G_{2yzi}\left(G_{fast\,i}-G_{2yzi}\right)}}\left[\operatorname{arctg}\left(1-\dfrac{\delta^*_{adh\,i}}{h_{2i}}\right)\sqrt{\dfrac{\nu_{0i}\left(G_{fast\,i}-G_{2yzi}\right)}{G_{2yzi}}}-\operatorname{arctg}\left(1-\dfrac{\delta^*_{2i}+2\delta^*_{adh\,i}}{2h_{2i}}\right)\sqrt{\dfrac{\nu_{0i}\left(G_{fast\,i}-G_{2yzi}\right)}{G_{2yzi}}}\right]$
	$f_{0i}\neq 0$ $h_{1i}+h_{2i}\le\dfrac{\delta^*_{2i}}{2}$	$\Pi_{adh\,xzi}=\Pi_{adh\,yzi}=\dfrac{\delta^*_{adh\,i}}{G_{fast\,i}\nu_{0i}+G_{adh\,i}\left(1-\nu_{0i}\right)}=\dfrac{\delta^*_{adh\,i}}{G_{adh\,i}+\nu_{0i}\left(G_{fast\,i}-G_{adh\,i}\right)}$; $\Pi_{2xzi}=\dfrac{\delta^*_{2i}+2\left(\delta^*_{adh\,i}-h_{1i}-h_{2i}\right)}{2G_{2xzi}}+\dfrac{h_{1i}-\delta^*_{adh\,i}}{G_{2xzi}+\nu_{0i}\left(G_{fast\,i}-G_{2xzi}\right)}+\dfrac{h_{2i}}{\sqrt{\nu_{0i}G_{2xzi}\left(G_{fast\,i}-G_{2xzi}\right)}}\operatorname{arctg}\sqrt{\dfrac{\nu_{0i}\left(G_{fast\,i}-G_{2xzi}\right)}{G_{2xzi}}}$; $\Pi_{2yzi}=\dfrac{\delta^*_{2i}+2\left(\delta^*_{adh\,i}-h_{1i}-h_{2i}\right)}{2G_{2yzi}}+\dfrac{h_{1i}-\delta^*_{adh\,i}}{G_{2yzi}+\nu_{0i}\left(G_{fast\,i}-G_{2yzi}\right)}+\dfrac{h_{2i}}{\sqrt{\nu_{0i}G_{2yzi}\left(G_{fast\,i}-G_{2yzi}\right)}}\operatorname{arctg}\sqrt{\dfrac{\nu_{0i}\left(G_{fast\,i}-G_{2yzi}\right)}{G_{2yzi}}}$

Table. 8 (continued)

Force connection type	Parameters value	Analytical dependences
	$f_{0i} \neq 0$ $h_{1i} + h_{2i} > \frac{\delta^*_{2i}}{2}$ $h_{1i} \leq \frac{\delta^*_{2i}}{2}$	$\Pi_{adh\,xzi}$, $\Pi_{adh\,yzi}$ – see previous scheme $\Pi_{2xzi} = \frac{h_{1i} - \delta^*_{adh\,i}}{G_{2xzi} + \nu_{0i}\left(G_{fast\,i} - G_{2xzi}\right)} + \frac{h_{2i}}{\sqrt{\nu_{0i} G_{2xzi}\left(G_{fast\,i} - G_{2xzi}\right)}}\left[\operatorname{arctg}\sqrt{\frac{\nu_{0i}\left(G_{fast\,i} - G_{2xzi}\right)}{G_{2xzi}}} - \operatorname{arctg}\frac{2\left(h_{1i} + h_{2i} - \delta^*_{adh\,i}\right) - \delta^*_{2i}}{2h_{2i}}\sqrt{\frac{\nu_{0i}\left(G_{fast\,i} - G_{2xzi}\right)}{G_{2xzi}}}\right]$ $\Pi_{2yzi} = \frac{h_{1i} - \delta^*_{adh\,i}}{G_{2yzi} + \nu_{0i}\left(G_{fast\,i} - G_{2yzi}\right)} + \frac{h_{2i}}{\sqrt{\nu_{0i} G_{2yzi}\left(G_{fast\,i} - G_{2yzi}\right)}}\left[\operatorname{arctg}\sqrt{\frac{\nu_{0i}\left(G_{fast\,i} - G_{2yzi}\right)}{G_{2yzi}}} - \operatorname{arctg}\frac{2\left(h_{1i} + h_{2i} - \delta^*_{adh\,i}\right) - \delta^*_{2i}}{2h_{2i}}\sqrt{\frac{\nu_{0i}\left(G_{fast\,i} - G_{2yzi}\right)}{G_{2yzi}}}\right]$
	$f_{0i} \neq 0$ $h_{1i} > \frac{\delta^*_{2i}}{2}$	$\Pi_{adh\,xzi}$, $\Pi_{adh\,yzi}$ – see previous scheme $\Pi_{2xzi} = \frac{\delta^*_{2i}}{2\left[G_{2xzi} + \nu_{0i}\left(G_{fast\,i} - G_{2xzi}\right)\right]}$; $\Pi_{2yzi} = \frac{\delta^*_{2i}}{2\left[G_{2yzi} + \nu_{0i}\left(G_{fast\,i} - G_{2yzi}\right)\right]}$
	$f_{0i} = 0$	$Ï_{1xzi} = \frac{\delta^*_{1i}}{2G_{1xzi}}$; $Ï_{1yzi} = \frac{\delta^*_{1i}}{2G_{1yzi}}$
	$f_{0i} \neq 0$ $G_{fast\,i} = G_{1xz}$ $G_{fast\,i} = G_{1yz}$	$\Pi_{1xzi} = \frac{\delta^*_{1i}}{2G_{fast\,i}}$; $\Pi_{1yzi} = \frac{\delta^*_{1i}}{2G_{fast\,i}}$
	$f_{0i} \neq 0$ $h_{4i} = 0$ $h_{3i} \leq \frac{\delta^*_{1i}}{2}$	$\Pi_{1xzi} = \frac{h_{3i}}{G_{1xzi} + \nu_{0i}\left(G_{fast\,i} - G_{1xzi}\right)} + \frac{\delta^*_{1i} - 2h_{3i}}{2G_{1xzi}}$; $\Pi_{1yzi} = \frac{h_{3i}}{G_{1yzi} + \nu_{0i}\left(G_{fast\,i} - G_{1yzi}\right)} + \frac{\delta^*_{1i} - 2h_{3i}}{2G_{1yzi}}$
	$f_{0i} \neq 0$ $h_{4i} = 0$ $h_{3i} > \frac{\delta^*_{1i}}{2}$	$\Pi_{1xzi} = \frac{\delta^*_{1i}}{2\left[G_{1xzi} + \nu_{0i}\left(G_{fast\,i} - G_{1xzi}\right)\right]}$; $\Pi_{1yzi} = \frac{\delta^*_{1i}}{2\left[G_{1yzi} + \nu_{0i}\left(G_{fast\,i} - G_{1yzi}\right)\right]}$

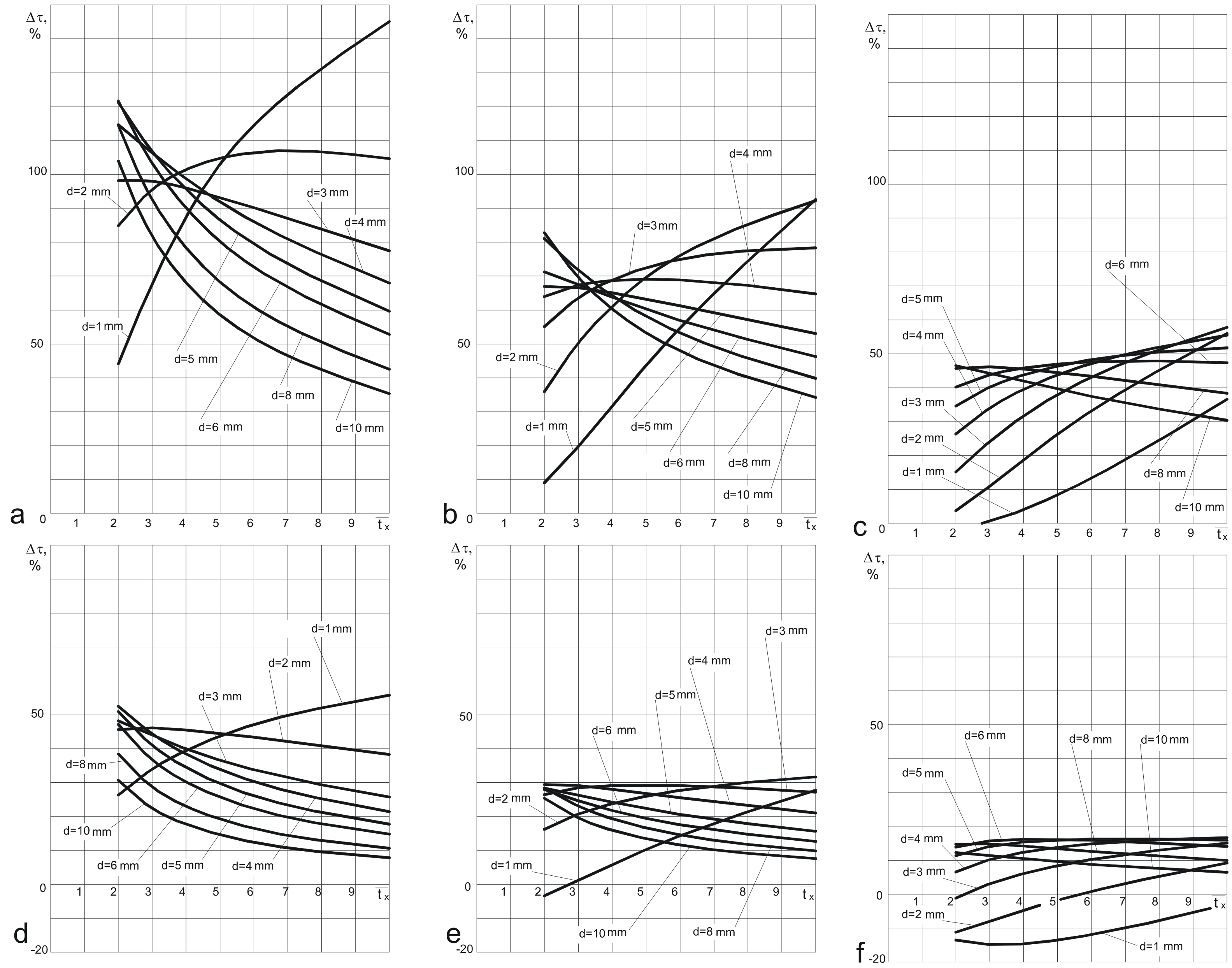


Fig. 33 – Relative deviation of maximum stress at application KhAI model (fastener is made of 3 0ХГСА)

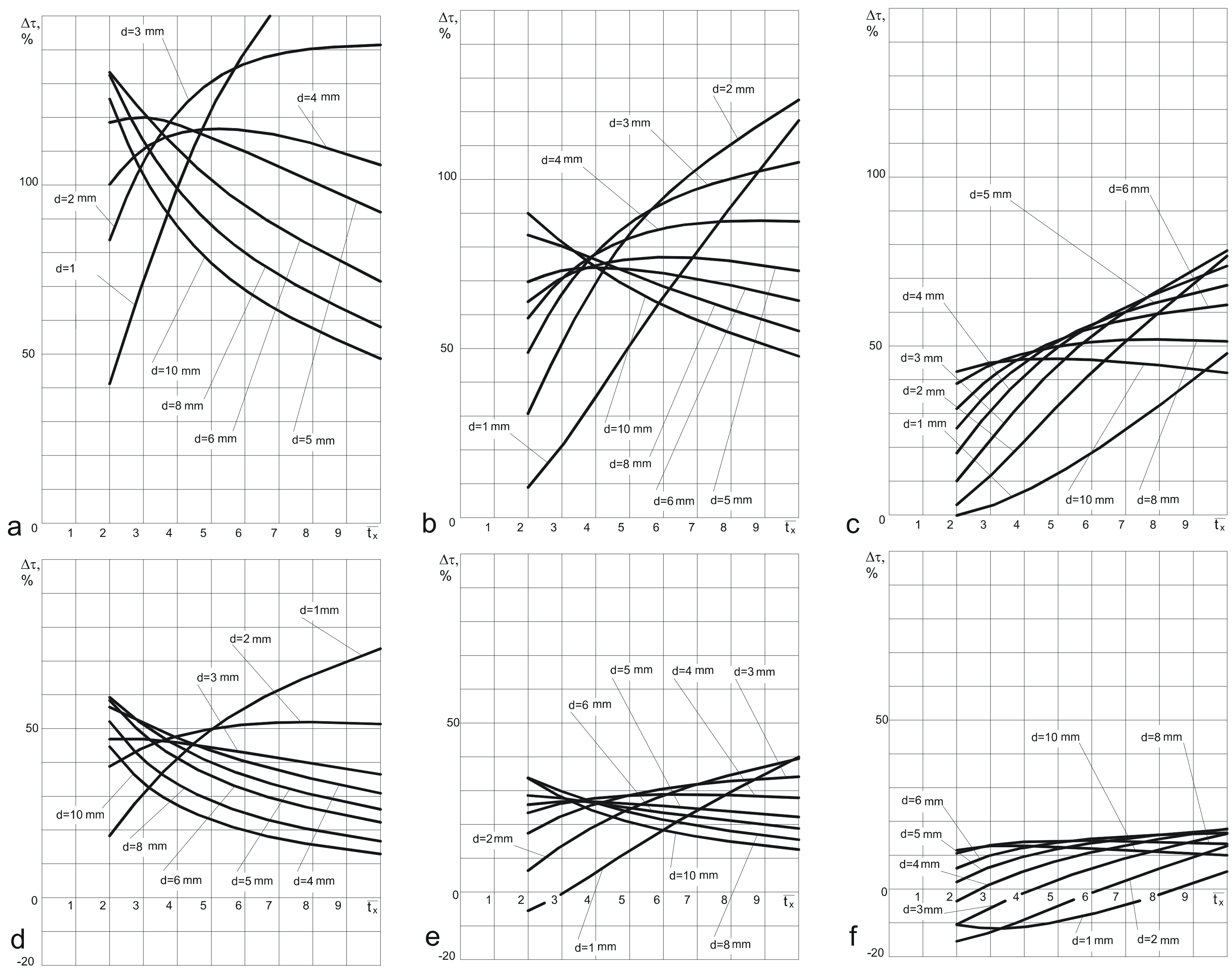


Fig. 34 – Relative deviation of maximum stress at application KhAI model (fastener is made of B96)

Section 8. Working out mathematical model of interaction micro-fastener embedded to composite with composite components

Methods, Assumptions, and Procedures

Process of embedding lateral micro-fasteners to uncured composite article is accompanied with moving reinforcing fibers apart and fibers warping at saving fibers structure integrity (fig. 35). This phenomenon brings about following sequences:

– at first, occurred fiber curvature changes elastic and strength properties of composite at micro-level, but designer needs to know exact value of properties for conduction correct stress-strain analysis of entire joint

– secondly, “co-current” zone consisting of pure resin only occurs near embedded micro-fasteners, as result fiber volume fraction at this zone changes from theoretical one and degree of composite physical and mechanical properties anisotropy (necessary for joint strength analysis at micro-level) becomes more significant.

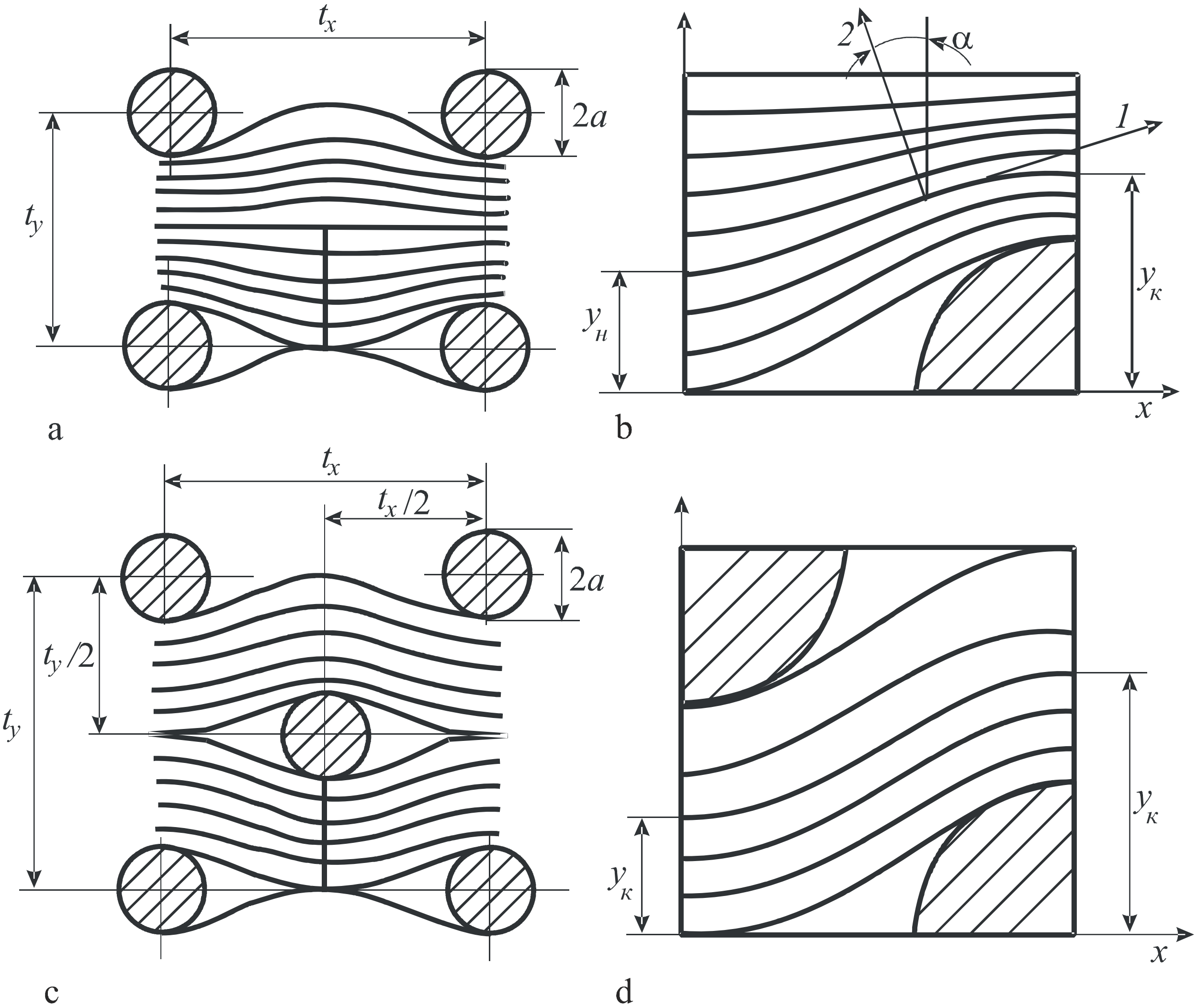


Fig. 35 Analytical scheme of interaction between micro-fasteners and composite fibers

Because of regularity of micro-fasteners installation (rectangular scheme (fig. 35, a) and chess scheme (fig. 35, c)) it is possible to distinguish representative structural elements for analysis of mentioned phenomenon. Following assumptions can be used for analysis [42, 43]:

– fibers are considered to be elastic rods, their diameter is relatively low comparing with micro-

fasteners spacing so technical theory of beam bending can be applied for analysis;

– micro-fasteners are considered to be very rigid;

– composite layers stay flat during micro-fasteners embedding;

– composite possesses initial (design) structure at section $x = 0$ (for rectangular micro-fasteners distribution) (see fig. 35, b);

– fibers are distributed uniformly inside composite volume at sections $x = 0$ and $x = t_x / 2$.

8.1 Rectangular fasteners arrangement

Last assumption permits to obtain dependence for determination coordinate y_K at known coordinate y_H:

$$y_K = \frac{a}{2} + y_H\left(1 - \frac{a}{t_y}\right). \tag{57}$$

Equation of fiber elastic line (beam elastic line) can be written as [44]:

$$\mathrm{W} = \mathrm{b}\mathrm{x}^3 + \mathrm{c}\mathrm{x}^2 + \mathrm{d}\mathrm{x} + \mathrm{l}, \tag{58}$$

coefficients of equation are defined from boundary conditions:

- at $x = 0$ $\quad W = y_H; \quad W' = 0;$

- at $x = \dfrac{t_x}{2}$ $\quad W = y_k; \quad W' = 0.$

Solution of system equations in terms of shape coefficients (58) gives

$$W = y_H + \frac{4x^2(y_k - y_H)}{t_x^2}\left(3 - 4\frac{x}{t_x}\right). \tag{59}$$

Inserting (57) to this expression we can obtain general equation of elastic line for any fiber with initial coordinate y_H:

$$W = y_H + \frac{2ax^2}{t_x^2}\left(3 - 4\frac{x}{t_x}\right)\left(1 - 2\frac{y_H}{t_y}\right). \tag{60}$$

Tangent of fiber slope refer to x axis is defined by formula

$$tg\alpha = W' = \frac{12ax}{t_x^2}\left(1 - 2\frac{x}{t_x}\right)\left(1 - 2\frac{y_H}{t_y}\right). \tag{61}$$

Deflection and angles of slope of edge fiber close to micro-fastener are defined as following:

$$W_0 = \frac{2ax^2}{t_x^2}\left(3 - 4\frac{x}{t_x}\right); \tag{62}$$

$$tg\alpha_0 = \frac{12ax}{t_x^2}\left(1 - 2\frac{x}{t_x}\right). \tag{63}$$

Therefore new directions of reinforcing fibers inside micro-volume at any point of composite are known. Fiber volume fraction in this micro-volume is defined as

$$\theta = \frac{\theta_H}{\left[1 - \frac{4ax^2}{t_x t_y}\left(3 - 4\frac{x}{t_x}\right)\right]\cos\alpha}, \tag{64}$$

where θ_H – initial fiber volume fraction in composite;

$$\cos\alpha = 1/\sqrt{1+\mathrm{tg}^2\alpha}\ . \tag{65}$$

Characteristics of any point of composite micro-volume can be estimated by known physical-mechanical properties of fibers and binder using reinforcing theory dependencies [4, 5, 6, 7, 8]. Then global properties of composite in axes of analyzed joint can be obtained by theory of laminated mediums [9, 10, 11, 7, 12]. This theory is very easy used for stress-strain analysis of composite near micro-fastener and for forces distribution along joint length.

To analyze global stress-strain of joint we have to use average elastic properties of composite which can be calculated by methods of solids mechanics at assumption that joint representative element boundaries stay straight after resulting load application (fig. 36, a). After some transformations one can obtain following expressions for determination average elasticity moduli at tension and shear and Poisson's ratios:

$$\frac{1}{E_{1aver}} = \frac{2}{t_x}\int_0^{t_x/2}\frac{dx}{E_1^*}; \qquad \frac{1}{E_{2aver}} = \frac{2}{t_y}\int_0^{t_y/2}\frac{dy}{E_2^*};$$

$$\frac{1}{G_{12aver}} = \frac{2}{t_x}\int_0^{t_x/2}\frac{dx}{G_{12}^*}; \tag{66}$$

$$\mu_{12aver} = \frac{2}{t_x}E_{1aver}\int_0^{t_x/2}\frac{\mu_{12}^*}{E_1^*}dx; \qquad \mu_{21aver} = \frac{2}{t_y}E_{2aver}\int_0^{t_y/2}\frac{\mu_{21}^*}{E_2^*}dy\ .$$

These formulas use following designations:

$$E_1^* = \frac{2}{t_y}\int_0^{t_y/2}E_1 dy; \quad E_2^* = \frac{2}{t_x}\int_0^{t_x/2}E_2 dx; \quad G_{12}^* = \frac{2}{t_y}\int_0^{t_y/2}G_{12} dy;$$

$$\mu_{12}^* = \frac{2}{t_y}\int_0^{t_y/2}\mu_{12} dy; \quad \mu_{21}^* = \frac{2}{t_x}\int_0^{t_x/2}\mu_{21} dx, \tag{67}$$

where E_1, E_2, G_{12}, μ_{12}, μ_{21} – elastic properties of composite at current point.

Taking into consideration that micro-fastener transmits load non-continuously along joint length through co-current zone of surrounding resin we can integrate equations (66) and (67) along y axis by two parts: from zero section up to W_0 (see formula (62)) and from W_0 up to $t_y/2$ for rectangular micro-fasteners arrangement.

Generally fiber volume fraction varies inside quite restricted range. That is why following dependences can be used for determination composite elastic properties at local points of considered zones:

$$E_{10} = E_{1H}\theta/\theta_H; \quad E_{20} = E_{2H}\theta/\theta_H;$$

$$G_{120} = G_{12H}\theta/\theta_H; \quad \mu_{120} = \mu_{12H}\theta/\theta_H, \tag{68}$$

where E_{1H}, E_{2H}, G_{12H}, μ_{12H} – initial elastic properties of composite at regular zone, θ, θ_H – current and initial fiber volume fraction.

Elastic properties defined by formulas (68) permit to calculate physical-mechanical properties of composite at current coordinate system (fig. 35, b axes 1, 2). That is why one has to use known formulas for axes rotation [13, 6, 14, 15, 16, 17, 12] to calculate elastic properties E_1, E_2, G_{12}, μ_{12} used in (67).

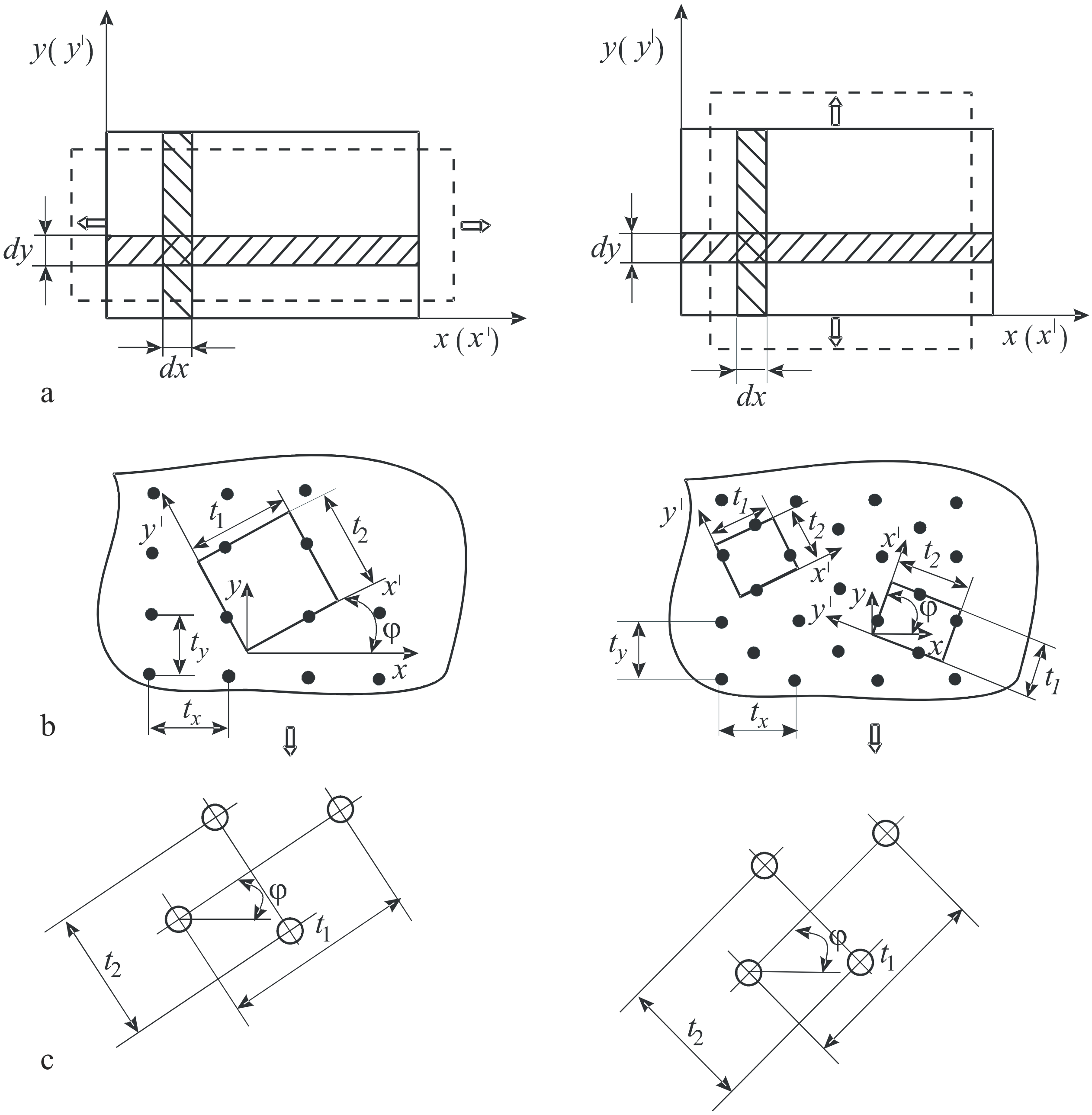


Fig. 36 Scheme for determination average values of composite elastic properties inside joint length

8.2 Chess micro-fasteners arrangement

Main difference of chess micro-fasteners from rectangular one is absence of section with initial (not-disturbed) composite properties. I.e. function of properties has periodic character having values not coincided with initial ones. Analysis of fig. 35, a, c shows composite has the same properties at sections $x = 0$ and $x = l/2$ (see fig. 35, d) which has one with rectangular fasteners arrangement at section $x = t_x/2$ (see fig. 35, b).

There is obvious relationship between origin $y_н$ and ending y_k coordinate of fiber

$$y_k = y_н + \frac{a}{2}. \tag{69}$$

Function of fiber deflection has view (65) and can be reduced as:

$$W = y_н + \frac{2ax^2}{t_x^2}\left(3 - 4\frac{x}{t_x}\right). \tag{70}$$

Following dependence for fiber slope tangent is valid:

$$tg\alpha = W' = \frac{12ax}{t_x^2}\left(1 - 2\frac{x}{t_x}\right). \tag{71}$$

New value of fiber volume fraction is defined as following

$$\theta = \frac{\theta^*}{\cos\alpha}, \tag{72}$$

where α is calculated by (65), and θ^* by (64) at $x = t_x / 2$ and $\alpha = 0$.

$$\theta^* = \theta_i / (1 - a / t_y). \tag{73}$$

After inserting (73) to (72) one can obtain

$$\theta = \theta_i / [\cos\alpha(1 - a / t_y)]. \tag{74}$$

We have to mention as conclusion that for micro-fasteners with rectangular or rhombic basement one has to use semi-length of joint side or length of diagonal directed along y axis instead of dimension a. Physical-mechanical properties of composite are variable through article thickness for pyramidal micro-fasteners. But character of this variation is described by the same formulas which are valid for cases where fibers origin plane stays flat (fibers deflection out of origin plane is absent).

Results and discussions

Exact values of composite properties at arbitrary fibers orientation can't be calculated because of difficulties of correct representative element selection. To obtain quite reliable engineering procedure for estimation composite layer elastic properties following considerations can be used (fig. 36, b, c): real arrangement of micro-fasteners is replaced by chess one at any point. Spacing between pins is defined by following dependences:

– for rectangular micro-fasteners arrangement

$$\begin{aligned} t_1 &= t_x \cos\varphi + t_y \sin\varphi; \\ t_2 &= t_x \sin\varphi + t_y \cos\varphi; \end{aligned} \tag{75}$$

where φ – reinforcing angle of composite layer;

– for chess micro-fasteners arrangement:

$$\text{at} \quad \varphi < \operatorname{arctg}\frac{t_y}{t_x} \qquad t_1 = t_x \cos\varphi; \quad t_2 = t_y \cos\varphi; \tag{76}$$

$$\text{at} \quad \varphi > \operatorname{arctg}\frac{t_y}{t_x} \qquad t_1 = t_y \sin\varphi; \quad t_2 = t_x \sin\varphi. \tag{77}$$

Conclusions

Average properties of composite with arbitrary reinforcing scheme are defined by following algorithm:

– new parameters (coordinates) of fasteners arrangement for each series of reinforcing angles φ_i are defined by formulas (75) – (77);

– new structural parameters of each composite layer (fiber volume fraction and fiber slope angle) are defined in coordinate system $x'y'$ (see fig. 36, b, c);

– elastic constants in local coordinate system 1,2 (see fig. 35) are defined by formulas (68);

– elastic properties of layer are calculated by rotation formulas; parameters $E_{1i}, E_{2i}, G_{12i}, \mu_{12i}$ (used in expression (67)) are defined at any point. Here one has to consider that all parameters and calculations are referred to coordinate system joined with reinforcing direction of each layer. Reinforcing angle at any point of composite are defined by formulas (61) or (71);

– average elastic characteristics of each layer are defined by dependences (66) considering formulas (67);

– average properties of entire composite package are calculated based on theory of laminated mediums [18, 19].

One has to note that for considered case integration process is conducted by means of integration limits replacement from t_x to t_1 and from t_y to t_2 in formulas (66) and (67). Therefore dependences for determination structural parameters of composite layer are the following

$$tg\alpha_{rec_i} = \frac{12ax'}{t_{1_i}^2}\left(1-2\frac{x'}{t_{1_i}}\right)\left(1-2\frac{y'_н}{t_{2_i}}\right);$$
$$tg\alpha_{ch_i} = \frac{12ax'}{t_{1_i}^2}\left(1-2\frac{x'}{t_{1_i}}\right); \qquad (78)$$

$$\left(\frac{\theta}{\theta_н}\right)_{reci} = \frac{1}{\cos\alpha_{reci}\left[1-\frac{4ax'^2}{t_{1_i}^2}\left(3-4\frac{x'}{t_{1_i}}\right)\right]}; \qquad (79)$$

$$\left(\frac{\theta}{\theta_н}\right)_{chi} = \frac{1}{\cos\alpha_{chi}\left(1-\frac{a}{t_2}\right)}, \qquad (80)$$

where indices "*rec*" и "*ch*" means rectangular and chess fasteners arrangement correspondingly.

Section 9. Analysis of micro-fasteners shape and arrangement influence on average elastic properties of composite at zone between fastener rows

Methods, Assumptions, and Procedures

To estimate degree of average composite properties variation due embedding to micro-fasteners to composite article two runs with carbon plastic КМУ-4э were done: for unidirectional one (fig. 37, 38) and for multiply-reinforced one (fig. 39 – 42) in which micro-fasteners were embedded according to rectangular and chess scheme [43]. Analysis of obtained dependences permits to make following conclusions:

- the most significant variation can be seen on elasticity modulus at the direction of initial reinforcement; moreover higher values of deviation are observed at chess pins arrangement (see fig. 37, 38). Analogous phenomenon occurs in multiple-reinforced composites;
- higher value of studying characteristic at regular zone higher its deviation at zones of micro-fasteners installation (see fig. 39 – 42);
- ration between micro-pins spacing influences on deformable composite properties according to dependence of fibers stacking angle variation change (see formulas (61) and (71));
- gradient of changing elasticity moduli and Poisson's ratios reduces with micro-fasteners volume fraction increasing.

Following designations are used at the figures 37 – 42:

$E_{x\,aver}$, $E_{y\,aver}$, $G_{xy\,aver}$, $\mu_{xy\,aver}$ – average values of elastic constants at the section of discretization with length t_x and width t_y;

E_x, E_y, G_{xy}, μ_{xy} – original composite properties;

$\nu_П$ – micro-fastener volume fraction, can be calculated by dependence

$$\nu_П = \frac{\pi a^2}{t_x t_y}. \qquad (81)$$

Results and Discussion

Results of analysis permit to make conclusion that correct embedding micro-pins to composite articles can cause significant stress redistribution along joint length at tension or compression loading. In the case of joining shells (mainly reinforced with angles ±45°) which generally has to withstand shear

loading it is necessary to take into consideration type and degree of shear modulus variation (see fig. 29 – 42) wchich in its turs defines shear stress distribution through micro-fasteners rows.

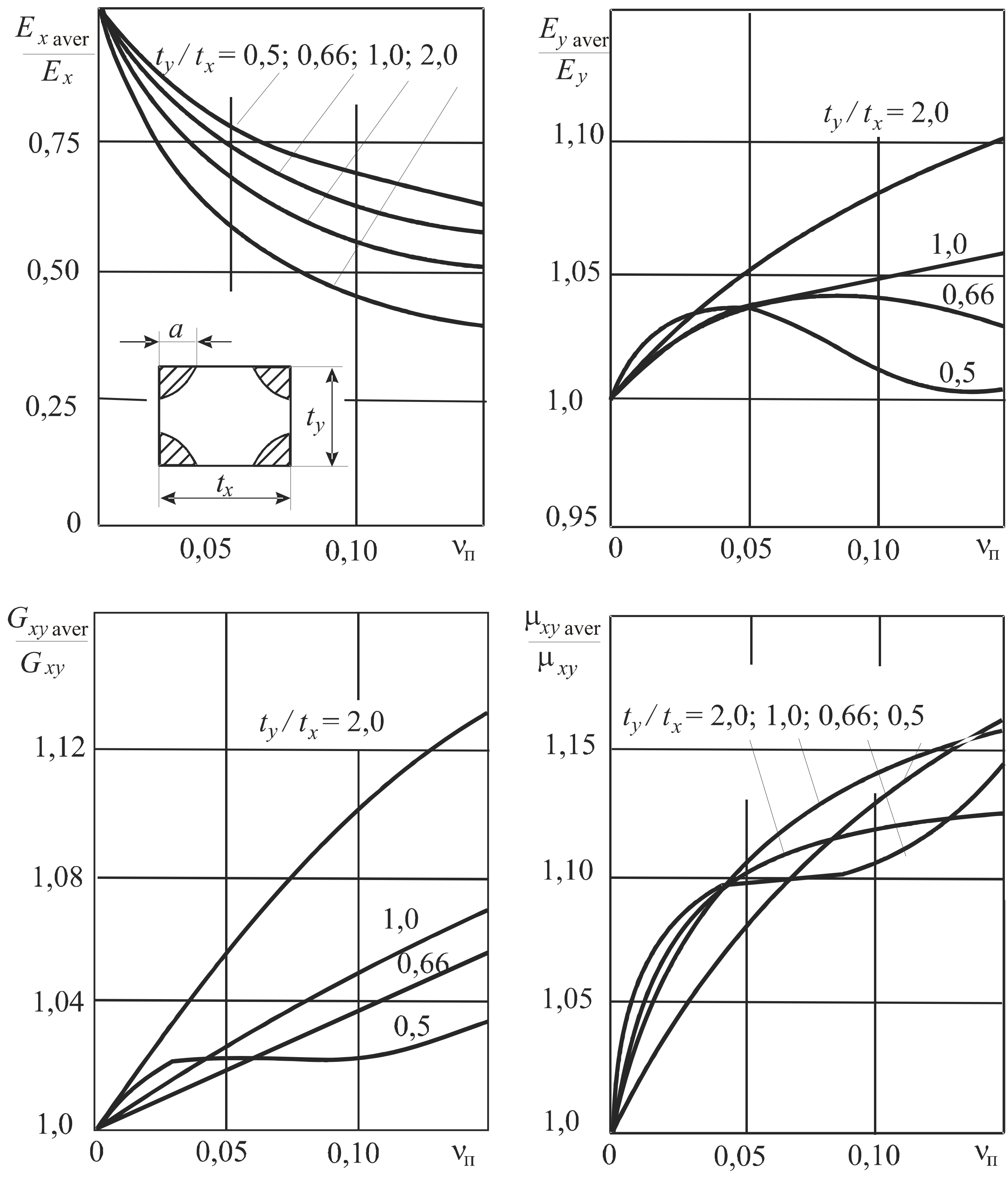


Fig. 37 Dependence of average elastic constants of composite mono-layer on micro-fastener parameters at their tetragonal arrangement

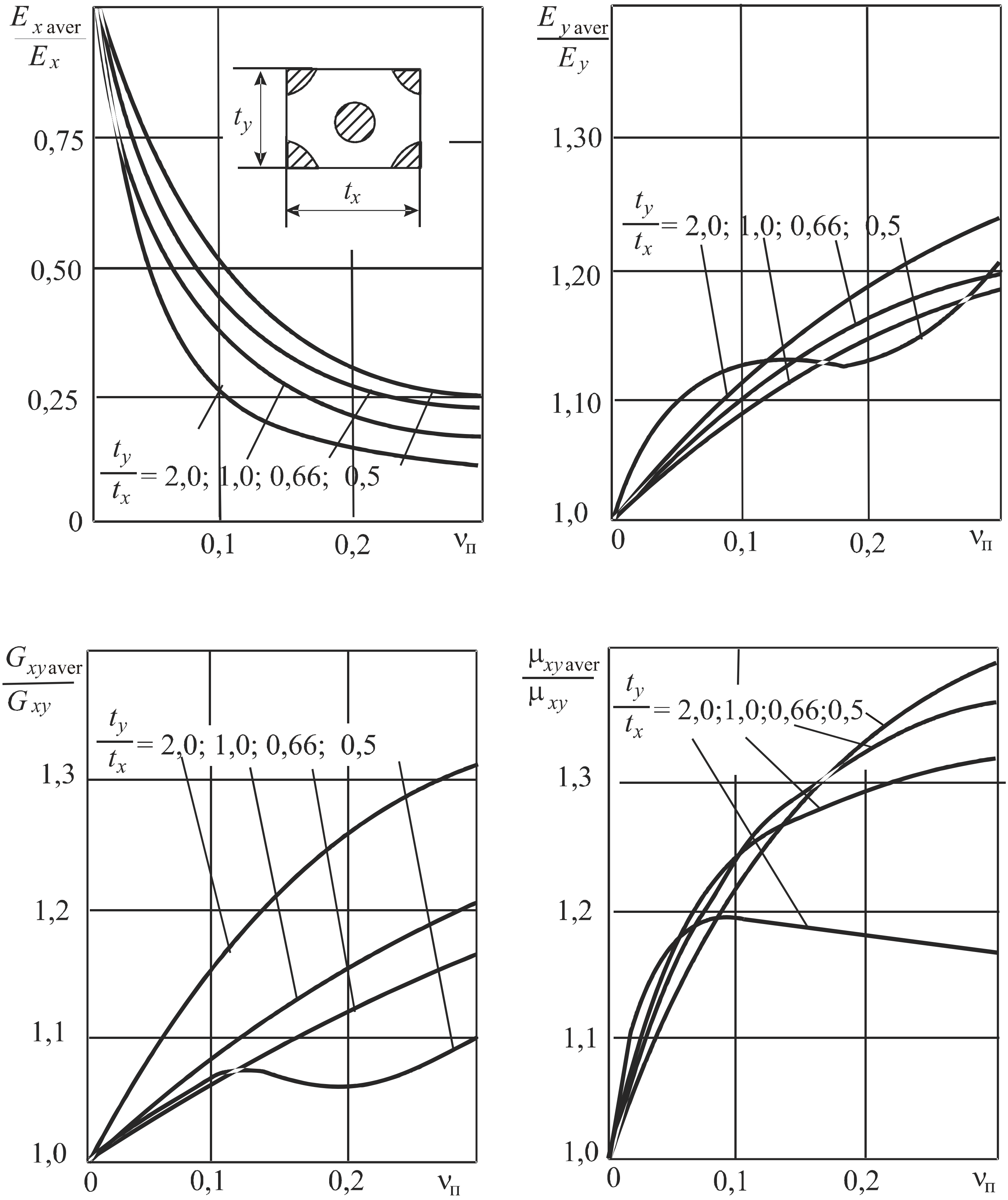


Fig. 38 Dependence of average elastic constants of composite mono-layer on micro-fastener parameters at their chess arrangement

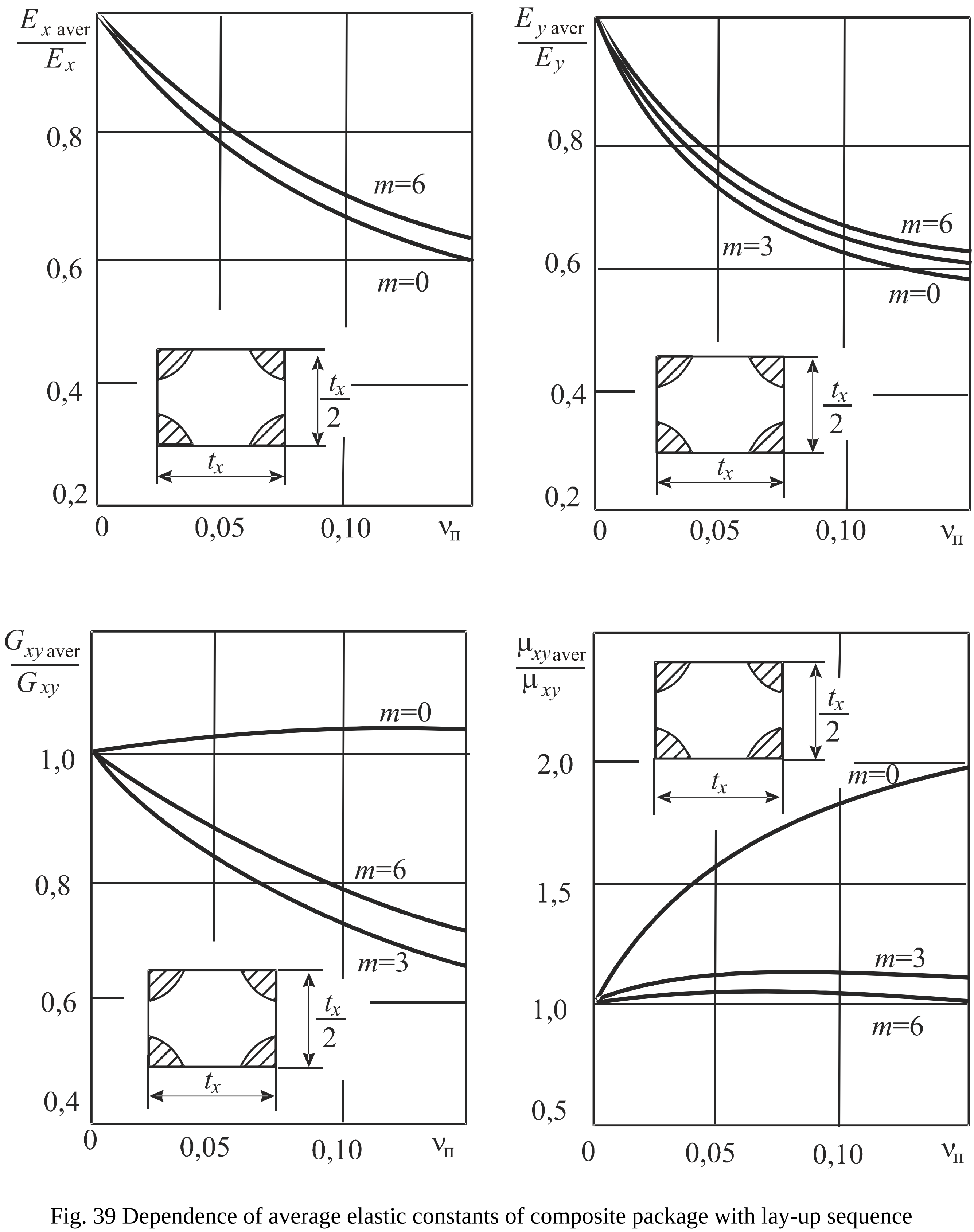


Fig. 39 Dependence of average elastic constants of composite package with lay-up sequence $\left[0^\circ_{18-2m}, \pm 45^\circ_m, 90^\circ_2\right]$ on micro-fastener parameters at their tetragonal arrangement

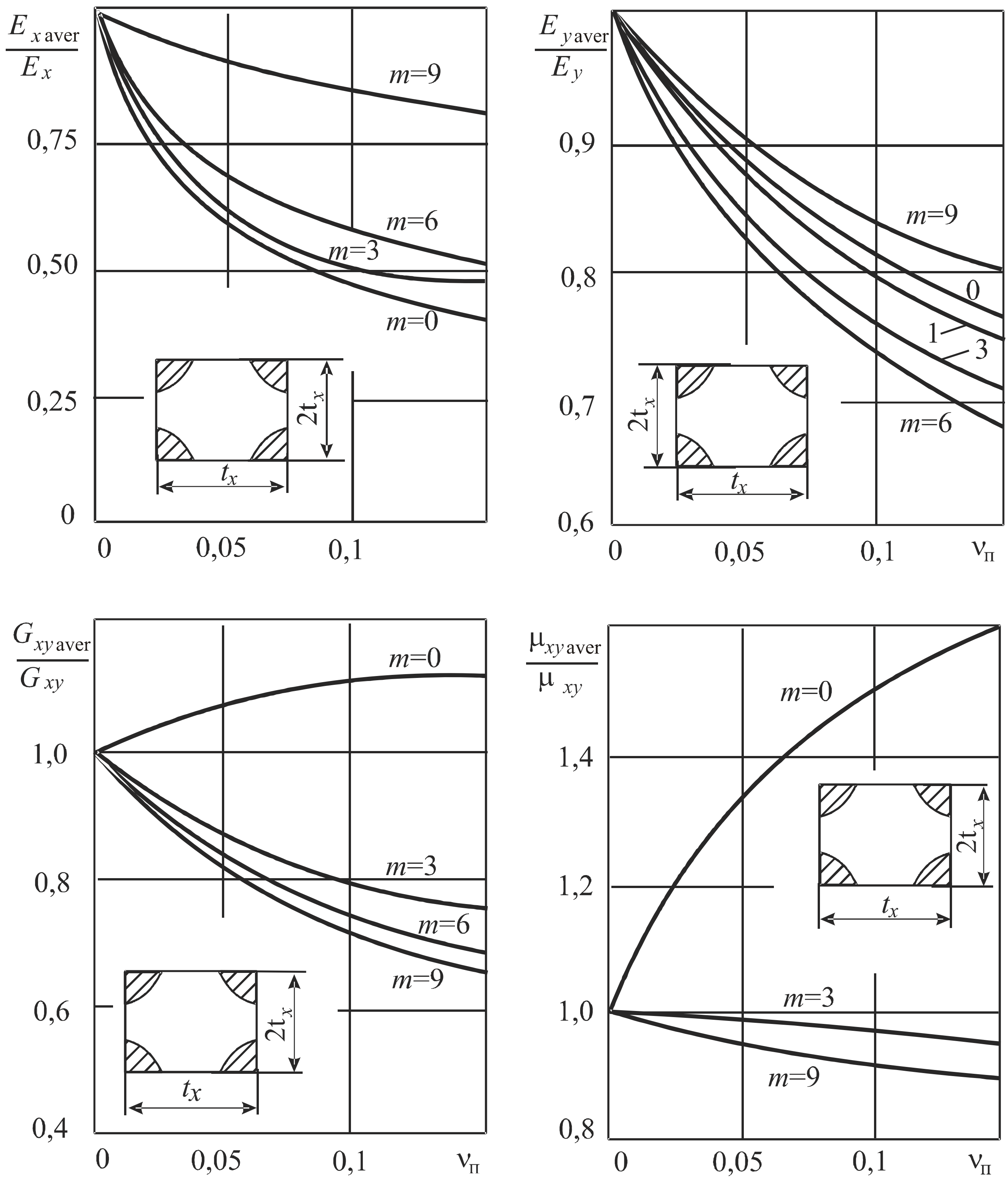


Fig. 40 Dependence of average elastic constants of composite package with lay-up sequence $\left[0^{\circ}_{18-2m}, \pm 45^{\circ}_{m}, 90^{\circ}_{2}\right]$ on micro-fastener parameters at their tetragonal arrangement

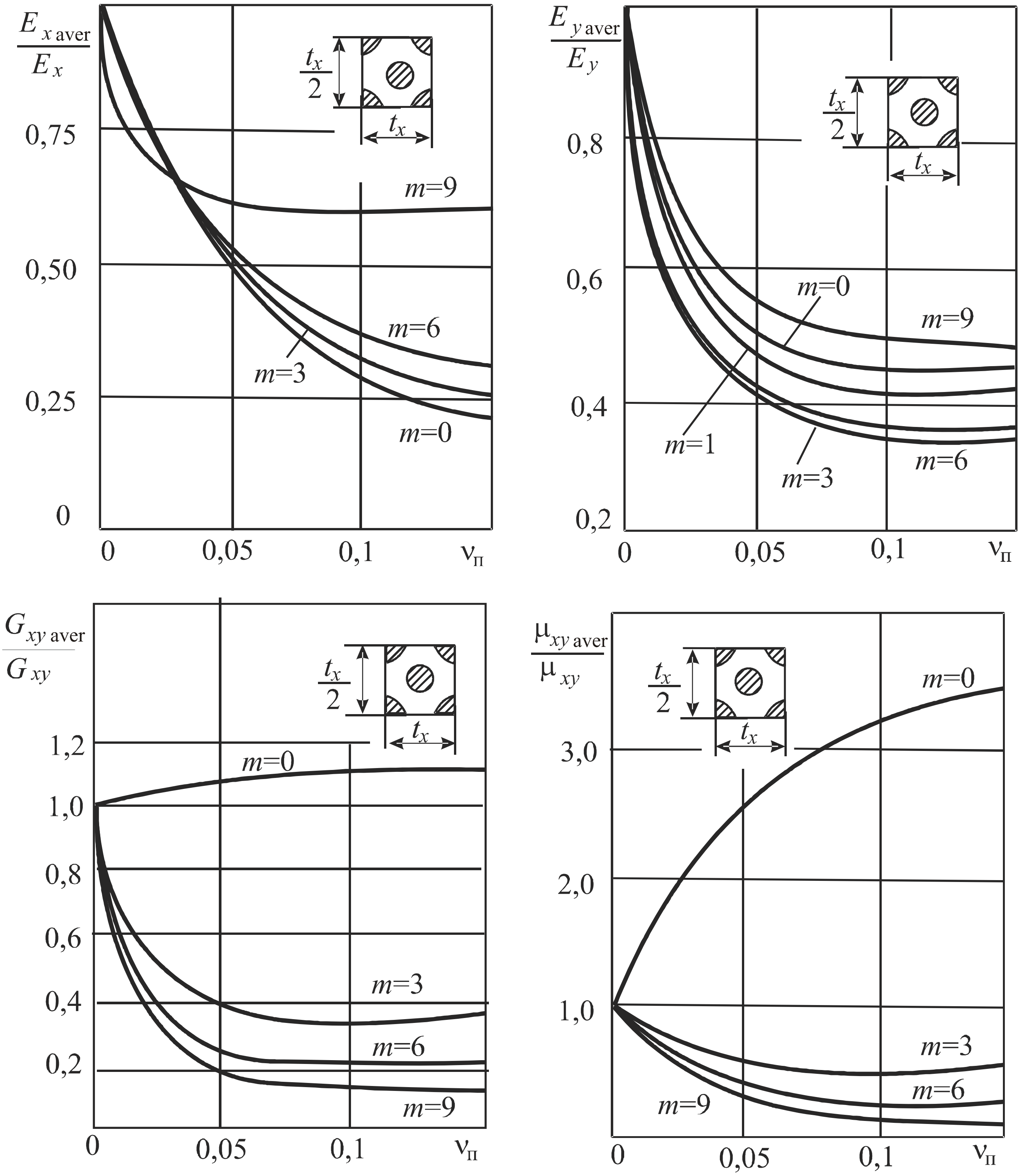


Fig. 41 Dependence of average elastic constants of composite package with lay-up sequence $\left[0^{\circ}_{18-2m}, \pm 45^{\circ}_{m}, 90^{\circ}_{2}\right]$ on micro-fastener parameters at their chess arrangement

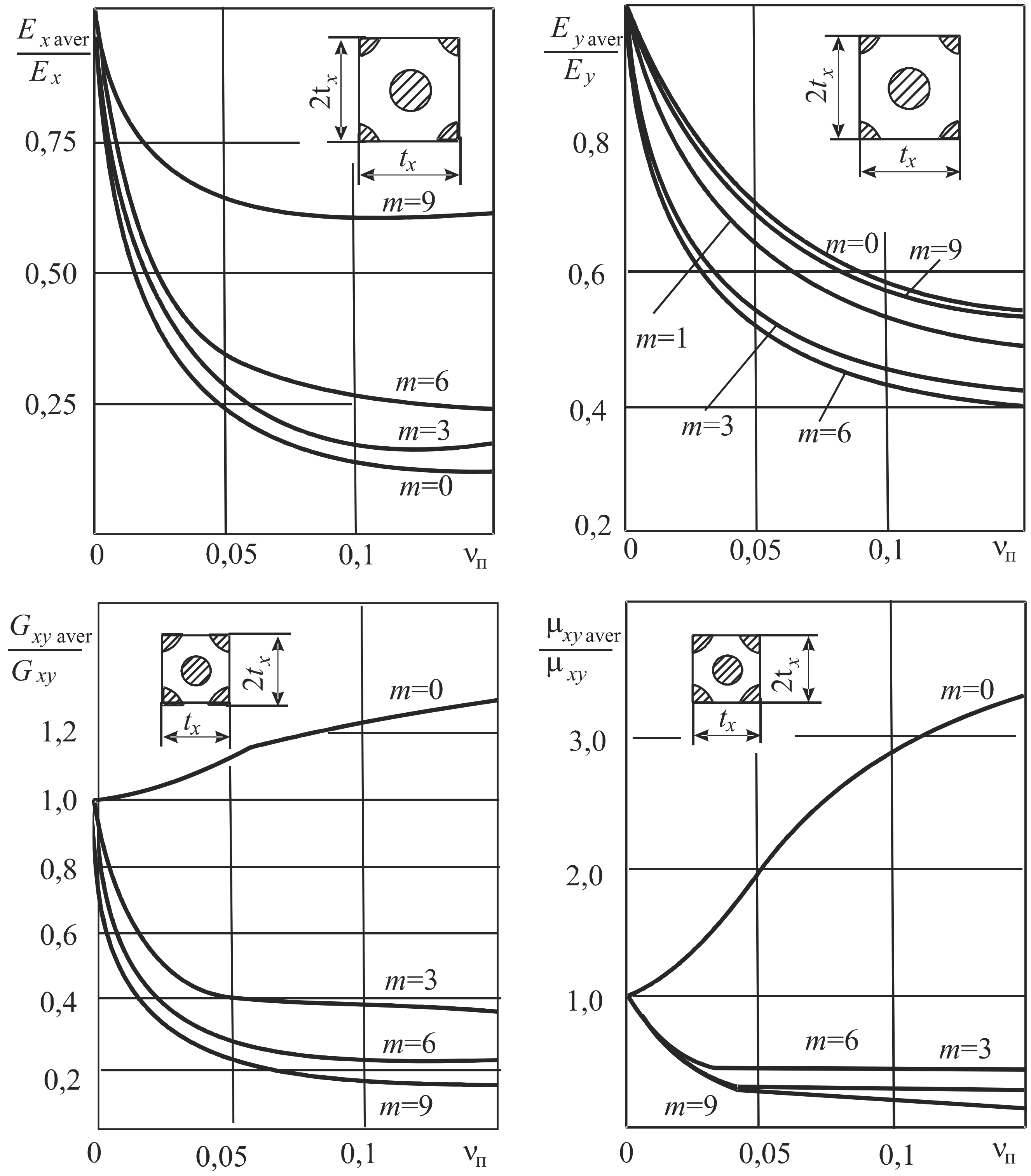


Fig. 42 Dependence of average elastic constants of composite package with lay-up sequence $\left[0^{\circ}_{18-2m}, \pm 45^{\circ}_{m}, 90^{\circ}_{2}\right]$ on micro-fastener parameters at their chess arrangement

Conclusions

Model of interaction between micro-fasteners and laminated composite fibers was developed. Practical dependencies permitting to determine current value of fiber slope, fiber volume fraction and as consequence elastic composite properties are worked out;

Studying influence of micro-pins embedded to composite article on composite elastic properties variation is done. Influence of such parameters as relative fastener arrangement spacing, arrangement scheme and composite layers stacking sequence on current composite properties is analyzed.

General conclusions

1. Conventional methods of joining (mechanical, adhesive and their combination) has practically reached their potential load-carrying abilities for the case of joining articles made of advanced materials. That is why further increasing of composite joint load-carrying ability draws crucial expenses. Analysis of prospective joining methods has revealed possibility of low-diameter fasteners application both in pure state and in combination with conventional joining methods. But this new method of joining is not well-studied yet, therefore special investigations were conducted and following results are obtained;
2. For reliable estimation of joining method quality special technique based on analysis of joint original margin load-carrying ability was worked out. Using this technique comparative estimation of original margin load-carrying ability of both conventional joining methods and joining by means of cylindrical micro-pins was conducted and fields of their primary application were advised;
3. Unified analysis scheme of joint with micro-fasteners was worked out. This scheme is based on one-dimensional joint model using method of physical discretizaton. Systems of solving equations for cases of normal, shear and lateral forces (appeared due to restricted thermal and Poisson's deformations in joining articles) transferring were derived based on mentioned analysis scheme). Dependences for determination compliance of joining articles and combined joining layer. Practical formulas for conduction joint elements stress-strain analysis were found;
4. Results of analysis according to developed universal model were compared with ones calculated by analytical model (for adhesive), discrete model recommended by TsAGI and numerical model based on FEA. Quite well convergence of analysis results by different models is found for mechanical joints and necessary discretization degree for adhesive joint (not less than 20 sections on each 100 of joint length);
5. To select well-grounded model of joining layer separate studying of joining layer compliance influence on its stress-strain state and especially on maximum stress level was conducted. This studying covered adhesive joint and mechanical joint at the same level of joining articles rigidity. It was established that in case of exceeding joint original margin load-carrying ability allowable relative variation of joining layer compliance doesn't depend on selected method of joining. Specific conditions at which non-uniformity of external loading distribution through joint length can be neglected;
6. Model for determination compliance of micro-fasteners having different top view shape and straight-linear generating line is worked out. Comparison of results obtained by suggested model with experimental ones is done, moreover strong reduction of micro-fasteners compliance out of allowable range was found. Therefore maximum stress is overestimated and restriction of suggested model application field can be required. Thus researches in this direction has to be continued;
7. Model of interaction between micro-fasteners and laminated composite fibers was developed. Practical dependencies permitting to determine current value of fiber slope, fiber volume fraction and as consequence elastic composite properties are worked out;
8. Studying influence of micro-pins embedded to composite article on composite elastic properties variation is done. Influence of such parameters as relative fastener arrangement spacing, arrangement scheme and composite layers stacking sequence on current composite properties is analyzed.

List of Symbols, Abbreviations, and Acronyms

AS– analysis scheme;
SMS– structural and manufacturing solutions;
D16T– aluminum alloy;
КМУ-4э– grade of carbon plastic;
k_1, k_2, k_3 – stress concentration factors;
$\tau_{\text{в fastener}}$ – ultimate fastener strength, MPa;
$\tau_{\text{в adhesive}}$ – adhesive shear strength, MPa;
L – joint length, mm;
t, t_x, t_y – fastener relative spacing or section length;
k_{red} – strength near opening reduction coefficient;
$\sigma_{bearing}$ – material bearing strength, MPa;
$E_{fastener}$– fastener elasticity modulus< GPa;
(d or $d_{fastener}$) – fastener diameter, mm;
i– ordinary number of row in multi-row joint or number of considered section;
j– ordinary number of individual fastener in *i*-th row;
N_{1i}, N_{2i} – internal forces, acting in first and second joining articles correspondingly at *i*–th section;
P_{1i}, P_{2i} – external forces applied to *i*–th node in first and second articles correspondingly;
Q_i – internal force, taking by *i*–th row of force connections;
α^*_{1xi}, α^*_{2xi} – average thermal linear expansion coefficients of articles material at definite section;
ΔT_1, ΔT_2 – temperature difference (is equal to difference between assembling temperature and operation temperature);
n – quantity of equations;
n_{1xi}, n_{2xi} – articles compliance along x axis (along joint length);
n_{3xi} – compliance of force connections row;
S_{yi} – force transmitted by *i*-th row;
Π_{1xyi}, Π_{2xyi} – shear compliance of articles in their plane;
Π_{3xyi} – compliance of force connections row along y-axis;
Δ_i – entire deformation of force connection row;
Δ_{ij} – deformation of *j*-th type component;
Q_{xi}, Q_{xij} – shear forces applied to entire *i*-th connection row and individually to *j*-th component in this *i*-th row;
Π_{3xzi}, Π_{3xzij} – correspondent shear compliance of unit area of entire connection row and component of *j*-th type in the plane *xz*;
f_{ij} – area of connection component of *j*-th type in *i*-th row;
f_i – area of *i*-th force connection row in the joining plane;
Π_{3yzi}, Π_{3yzij} – correspondingly specific shear compliance of entire connection row and *j*-th component type in the plane *yz*;
t_{yil} – discreetization spacing in longitudinal and lateral direction within *i*-th row at *l*-th section;
$\delta_1(x)$, $\delta_2(x)$ – articles thickness in analyzing section;
$\bar{\tau}_{yzi}$ – shear stress in joining layer due to shear force transferring;

$\bar{\tau}^{*}_{yzij}$ – shear stress in joining layer due to normal deformation restriction in lateral direction;
ξ– deviation of studying parameter (stress etc.);
α– current stacking angle of a fiber;
W– fiber deflection;
θ– fiber volume fraction;
μ, μ_{xy}, μ_{yx}– Poisson's ratio;
$E_{x\ aver}$, $E_{y\ aver}$, $G_{xy\ aver}$, $\mu_{xy\ aver}$– average values of composite elastic properties;
ν_{Π}– fastener volume fraction.

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